A Personalized Feedback Mechanism based on Bounded Confidence to Support Consensus Reaching in Group Decision Making

Quanbo Zha, Yucheng Dong, Hengjie Zhang, Francisco Chiclana, and Enrique Herrera-Viedma

***Abstract*—Different feedback mechanisms have been reported in consensus reaching models to provide advices for preference adjustment to assist decision makers to improve their consensus levels. However, the existing feedback mechanisms seldom discuss the willingness of decision makers to accept the advices they receive. In the discipline of opinion dynamics, the bounded confidence model justifies well that in the process of interaction decision makers take only into account the preferences that differ from theirs not more than a certain confidence level. Inspired by this idea, this paper proposes a new consensus reaching model with personalized feedback mechanism to help decision makers with bounded confidences in achieving consensus. Specifically, the personalized feedback mechanism produces more acceptable advices in the two cases where bounded confidences are known or unknown. Finally, numerical and simulation analysis are presented to explore the effectiveness of the proposed model in reaching consensus.**

***Index Terms*— Group decision making, Preference relation, Soft consensus, Bounded confidence, Personalized feedback mechanism.**

I. Introduction

Group decision making (GDM) is a scenario where a group of decision makers bring together their preferences to arrive at a common solution from a set of feasible solutions or alternatives [18], [19], [20], [21], [29], [31], [35], [38], [45]. To ensure that all decision makers will accept the GDM solution, two processes are usually included in the resolution process [2], [14], [22], [26], [36], [37]: the consensus reaching process (CRP) and the selection process. The purpose of a CRP is to improve the level of consensus among the decision makers in the group. Once an acceptable consensus level is reached, the selection process is activated to obtain a ranking of the alternatives from the collective preference that result from fusing all the preferences of decision makers.

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In a CRP, the feedback mechanism is a key phase in building consensus, because it provides adjustment advice in form of consensus rules to help decision makers to improve their consensus levels, which usually are of two types: (1) the identification and direction rules [26], [27], and (2) the minimum adjustments or cost rules [48]. The first one is used to identify the decision makers who require to modify their preferences and are targeted for receiving appropriate direction of adjustment of their preferences. Zhang et al. [51] proposed an identification and direction rules based consensus model that also implemented individual consistency levels; Herrera-Viedma et al. [28] used the identification and direction rules to build consensus in GDM with multigranular linguistic preference relations, while Wu et al. [42] applied the identification and direction rules to facilitate consensus in social network GDM with trust propagation. The second type of rules is utilized to minimize adjustments and cost in the process of achieving consensus in GDM. Zhang et al. [47] proposed a 2-rank CRP using the minimum adjustments in their feedback mechanism in a multi-granular linguistic context; Ben-Arieh and Easton [1] focused their efforts in achieving a minimum cost consensus in multi-criteria decision problem; Wu et al. [41] presented a minimum adjustment cost feedback mechanism to achieve a consensus in social network GDM, while Zhang et al. [50] proposed a consensus model to minimize information loss in GDM with heterogeneous preference structures.

In order to achieve consensus, most of the existing CRP feedback mechanisms advise the decision makers to adjust their preferences to value closer to the collective preference [10], [12], [15], [16], [25], [43], although they seldom consider the willingness of the decision makers to accept these advices. Recently, Li et al. [34] investigated this issue in a CRP via the use of the personalized individual semantics method to improve the willingness of decision makers. In the field of opinion dynamics [5], [13], [17], [39], the psychological factors of accepting opinions of others have been extensively studied in the form of bounded confidence, where it is argued that a decision maker only takes into account those opinions that are not far from their own opinion more than a certain confidence level. Two bounded confidence based models, the Hegselmann-Krause model [23] and the Deffuant-Weisbuch model [11], [40], have attracted wide attention and research in the communities of complexity science, sociophysics and social simulation, which both depend on the basic idea of decision makers having bounded confidences, i.e., decision makers with bounded confidences would reject the advices in a CRP if their preferences are far from the advised ones, which may obviously lead to a failure in reaching consensus in GDM. This is contrary to the basic hypothesis in most of the existing CRPs, which assumes that decision makers will always accept the provided advices. To our knowledge, there are very few research studies on this issue on spite of being common and reasonable in consensus-based GDM. To address this gap, this paper proposes a new consensus reaching model by considering the individual willingness of accepting advice, which aims at helping decision makers with bounded confidences to reach consensus. The main contributions of the paper are as follows:

(1) We reveal that most existing CRPs ignore the individual willingness to accept feedback adjustment advice, which strongly influences the CRP. The influence of individual willingness on the CRP has been investigated using simulation analysis.

(2) The opinion dynamic bounded confidence model is an effective way (methodology) to investigate individual willingness in opinion interactions. Inspired by the basic idea of the bounded confidence model, a new personalized feedback mechanism based consensus reaching model that implements individual willingness regarding the acceptance of advices is proposed.

(3) The effect of the personalized feedback mechanism on the improvement of individual willingness to accept feedback adjustment advice is analyzed in detail. Notably, the bounded confidences of decision makers are unknown in some situations, and a learning algorithm is presented to estimate the unknown bounded confidences in the CRP.

Both numerical and simulation analyses are proposed to justify the effectiveness of our model on consensus reaching.

The rest of the paper is organized as follows. In Section II, the basic knowledge about the general framework of CRPs and the bounded confidence in opinion dynamics are introduced. In Section III, the CRP problem with bounded confidence is proposed, and its resolution framework based on a personalized feedback mechanism is presented. Section IV presents a numerical example to illustrate the use of the proposed model and a simulation analysis is carried out to analyze its effectiveness regarding the reaching of consensus. Finally, conclusions are drawn in Section V.

II. Preliminaries

The basic concepts and methods regarding a general consensus reaching framework and bounded confidence model are introduced in this section.

A. A general consensus reaching framework

A GDM problem is generally described as a decision situation where a group of decision makers $D=\{d_1,d_2,\cdots,d_m\}(m\geq 2)$ express their preferences on a set of alternatives $X=\{x_1,x_2,\cdots,x_n\}(n\geq 2)$ with the aim of achieving a collective solution. Additive preference relations (also called fuzzy preference relations) are widely used to provide and capture preferences within a GDM framework [6], [8], [30], [32]. Without loss of generality, in this paper we assume that each decision maker $d_k \in D$ expresses their preference over X using an additive preference relation, $P_k=(p_{ij}^k)_{n\times n}$, where $p_{ij}^k\in[0,1]$ denotes the preference degree of alternative x_i over x_j and $p_{ij}^k+p_{ji}^k=1$.

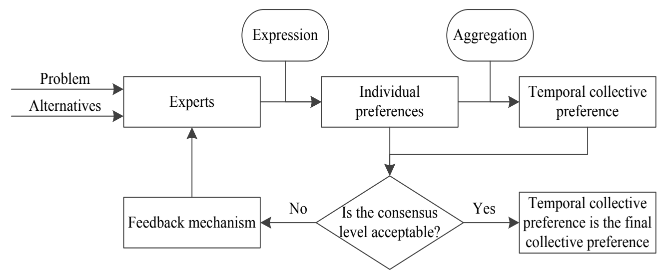

Fig.1. A general consensus reaching framework

A framework as shown in Fig. 1 is the essence procedure of most consensus reaching models, which includes two processes: CRP and selection process. Two are the key phases in the CRP for improving consensus level among decision makers. The first of these steps refers to the measuring of agreement/consensus among the group of decision makers, while the second one applies when the group consensus level is unacceptable and a feedback process to advise decision makers on how to improve their consensus is activated.

(1) *Consensus measure* quantifies the distances between decision makers' preferences and collective preference [7], [26]. Different aggregation operators have been developed to aggregate the preferences of decision makers to obtain the collective preference relation $P_c=(p_{ij}^c)_{n\times n}$. Among them, we have the weighted average (WA) and the ordered weighted average [46] operators. In this paper the WA operator is employed, and therefore the collective preferences would be obtained as follows:

$$p_{ij}^c = WA_w(p_{ij}^1,p_{ij}^2,\cdots,p_{ij}^m)=\sum_{k=1}^{m} w_k p_{ij}^k,\ \text{for } i,j=1,\cdots,n \qquad (1)$$

where w_k is a non-negative normalized weight associated to decision maker d_k, i.e., subject to the constraint $\sum_{k=1}^{m} w_k=1$. Using the Euclidean distance, the consensus level of decision maker d_k and the consensus level among all decision makers are defined, respectively, as

$$cl(d_k)=\frac{\sum_{i=1}^{n-1}\sum_{j=i+1}^{n}(1-|p_{ij}^k-p_{ij}^c|)}{n\cdot(n-1)/2} \qquad (2)$$

$$cl = \frac{\sum_{k=1}^{m} w_k cl(d_k)}{m} \quad (3)$$

The closer cl is to 1 the more agreement between the group of decision makers. If $cl \geq \mu$, the temporal collective preference will be the final collective preference, where $\mu \in [0,1]$ is a priori fixed consensus threshold (closer to 1 than 0). Otherwise, it can be argued that decision makers are in a state of unacceptable consensus level, and they will be advised to adjust their preferences to improve consensus level.

(2) *Feedback mechanism* aims to support decision makers to reach a higher consensus level, which is facilitated by the following two rules [3], [28]:

(a) *Identification rule* by which the decision maker(s) with the minimum consensus level as per Eq. (2) is(are) identified.

(b) *Direction rule* of recommended changes of preferences to the identified decision maker(s) d_k in step 2(a) are advised to get closer to the collective preference P_c with the aim of increasing their corresponding. consensus level, and consequently the group consensus level. The direction rules of recommended adjusted preference $\overline{p}_{ij}^k$ are:

$$\begin{cases} \overline{p}_{ij}^k \in [\min(p_{ij}^k, p_{ij}^c), \max(p_{ij}^k, p_{ij}^c)], & i \geq j \\ \overline{p}_{ij}^k = 1 - \overline{p}_{ji}^k, & i < j \end{cases} \quad (4)$$

Once the group consensus level is acceptable, i.e., a selection process to derive the rankings of alternatives from the collective preference will be activated. Using the WA operator, the value

$$Q_i = WA_\sigma(p_{i1}^c, p_{i2}^c, \cdots, p_{in}^c) = \sum_{j=1}^{n} \sigma_j \cdot p_{ij}^c \quad (5)$$

where $\{\sigma_1, \sigma_2, \cdots, \sigma_n\}$ are the weights associated to the alternatives with $\sum_{k=1}^{n} \sigma_k = 1$, can be interpreted as a value of dominance of alternative over the rest of alternatives [4], [9], [24]. The higher the evaluation value, the higher the ranking of alternative will be in the final ordering of the set of alternatives.

B. Bounded confidence

The bounded confidence model is a popular tool in the discipline of opinion dynamics that conveys the individual willingness of referring to the opinions of others [17], [23]. In this model, a decision maker will only take into account the opinions which differ from his/her opinion not more than a certain confidence level, which is briefly described below.

Let $P_a = (p_{ij}^a)_{n \times n}$ be the advice assigned to decision maker d_k, $P_k = (p_{ij}^k)_{n \times n}$ be his/her expressed preference, and $\varepsilon_k \in [0,1]$ be his/her bounded confidence. Then, the advice P_a will be accepted by decision maker d_k only if its distance D_{ka} from P_k is smaller than ε_k, i.e.,

$$D_{ka} \leq \varepsilon_k \quad (6)$$

with $D_{ka} \in [0,1]$ computed as follows:

$$D_{ka} = \frac{\sum_{i=1}^{n-1} \sum_{j=i+1}^{n} | p_{ij}^k - p_{ij}^a |}{n \cdot (n-1)/2} \quad (7)$$

III. Consensus Reaching Model with Bounded Confidence

In this section, the CRP problem with bounded confidence is proposed, and its resolution framework based on a personalized feedback mechanism is presented.

A. Framework of consensus reaching model with bounded confidence

The general CRP presented in Section 2 assumes that the advice derived from the feedback mechanism are all agreeable for the identified decision makers, and that they will always accept them. However, in reality decision makers would oppose to adjust their preferences if they disagree with the recommended advice. Therefore, the willingness of decision makers to accept the advice should be considered in CRP, which supports the worthiness of considering the CRP problem within the bounded confidence framework. Consequently, decision makers will judge whether they will accept these advices based on their own bounded confidences, and the CRP problem will deal with how to obtain a consensus based solution taking into account their bounded confidence levels. The proposed bounded confidence CRP framework is shown in Fig. 2. The selection process is assumed the same one in Section 2, and the main difference with the previous CRP framework of Fig. 1 is the Step 2 *Personalized feedback mechanism*, which will provide personalized advices to the identified decision makers based on their known and unknown bounded confidences; additionally, a learning algorithm is included to estimate the unknown bounded confidences for providing more acceptable advices; this is described in detail in the next section.

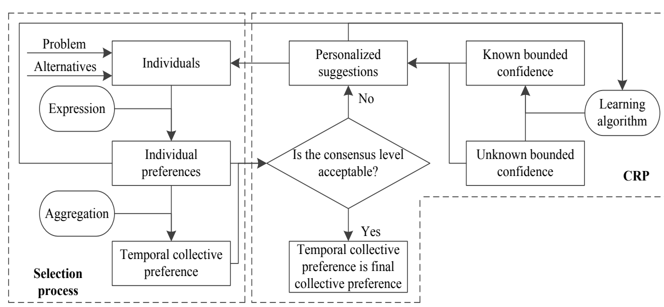

Fig. 2. The CRP framework considered bounded confidence

B. A personalized feedback mechanism

In the general CRP framework described in Section 2.1, the collective preference relation P_c is directly employed to produce the step (b) feedback mechanism advices as per expression (4). However, the identified decision maker(s) in Step (a) of the feedback mechanism might not accept the collective preferences if they distance from their own preferences more than their corresponding bounded confidences, which might impact the reaching of consensus in this general CRP framework. In order to overcome this limitation, a personalized feedback mechanism based on the bounded confidence is proposed, which is argued will favour decision makers' willingness to accept the personalized advices, which will lead to increasing the group consensus. The personalized feedback mechanism will still consist of two rules: (1) the *personalized identification rule* (PIR), which is identical to the rule described in Section 2.1 to identify the decision maker(s) with minimum consensus level, and (2) the *personalized direction rule* (PDR), which provides advices to increase the group consensus level based on known or unknown bounded confidence. This second new rule is elaborated below.

Suppose that PIR identifies decision maker d_k. Let ε_k be his/her bounded confidence, P_a be the feedback mechanism advice presented to decision maker d_k, and D_{ka} be the distance between the advice P_a and decision maker d_k own preference relation P_k. Similarly, let D_{kc} be the distance between the collective preference relation P_c and P_k. Below, two possible cases regarding knowledge of the bounded confidence are discussed.

(1) *Bounded confidence is known.* A personalized feedback mechanism based on the known bounded confidence is proposed to provide the advice to the identified decision maker as shown in Fig. 3, which includes two forms of advice as follows:

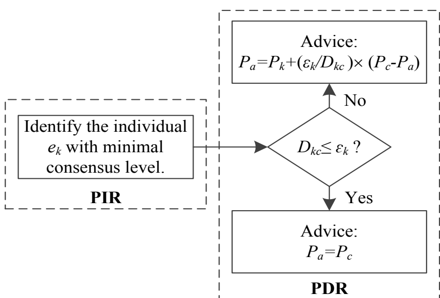

Fig.3. The framework of personalized feedback mechanism based on known bounded confidence

(a) If $D_{kc} \le \varepsilon_k$, the collective preference relation P_c will be within the bounded confidence of decision maker d_k and, consequently, he/she will be willing to accept it as his/her personalized advice, i.e., $P_a = P_c$. The adjustment direction is in this case will be the same as described in the Eq. (4).

(b) If $D_{kc} > \varepsilon_k$, then will not verify the bounded confidence of decision maker d_k, and the advice P_a provided to decision maker d_k will differ from P_k in order to satisfy $D_{ka} \le \varepsilon_k$ and being acceptable to decision maker d_k. It is noticed that when

$$P_a = P_k + \frac{\varepsilon_k}{D_{kc}}(P_c - P_k) \tag{8}$$

will satisfy

$$D_{ka} = \frac{\sum_{i=1}^{n-1}\sum_{j=i+1}^{n} \left| \frac{\varepsilon_k}{D_{kc}}(p_{ij}^c - p_{ij}^k) \right|}{n \cdot (n-1)/2} = \varepsilon_k .$$

Specifically, according to Eq. (8), we have

$$p_{ij}^a = \frac{D_{kc} - \varepsilon_k}{D_{kc}} p_{ij}^k + \frac{\varepsilon_k}{D_{kc}} p_{ij}^c \tag{9}$$

which guarantees that $p_{ij}^a + p_{ji}^a = 1$ and $p_{ij}^a \in [\min(p_{ij}^k, p_{ij}^c), \max(p_{ij}^k, p_{ij}^c)] \subset [0,1]$. Thus, to improve consensus level of the identified decision maker, the advised adjustment direction is

$$\begin{cases} \overline{p}_{ij}^k \in [\min(p_{ij}^k, p_{ij}^a), \max(p_{ij}^k, p_{ij}^a)], & i \ge j \\ \overline{p}_{ij}^k = 1 - \overline{p}_{ji}^k, & i < j \end{cases} \tag{10}$$

Notice that because of $p_{ij}^a \in [\min(p_{ij}^k, p_{ij}^c), \max(p_{ij}^k, p_{ij}^c)]$, this adjustment direction ensures that the adjusted preference $\overline{p}_{ij}^k$ will be closer to the collective preference p_{ij}^c than the preference it will replace.

(2) *Bounded confidence is unknown.* In this case, the bounded confidence of a decision maker will be estimated via an interval $[b_{k,\max}, b_{k,\min}]$, i.e., an identified unknown bounded confidence will verify. The initial estimated interval is set as [0,1]. Let θ be the bounded confidence threshold. If $b_{k,\max} - b_{k,\min} \le \theta$, then it can be argued that the estimation is accurate enough and there is no need to estimate the bounded confidence of decision maker d_k and the advice can be provided based on the accurate estimated interval. In this way, the personalized feedback mechanism shown in Fig. 4 based on the estimated bounded confidences is proposed to generate the advice:

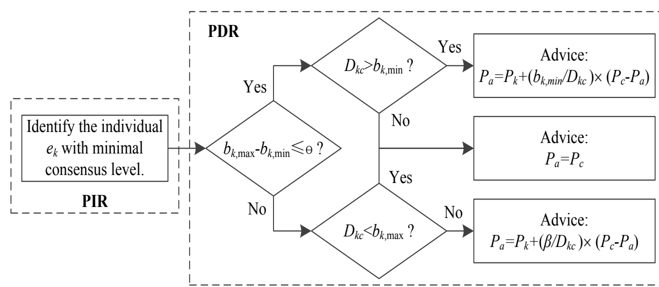

Fig.4. The framework of personalized feedback mechanism based on unknown bounded confidence

(a) When $b_{k,\max} - b_{k,\min} \le \theta$ and $D_{kc} > b_{k,\min}$, we have that the estimated interval is accurate enough but the identified decision maker d_k could decline the collective preference relation P_c as the recommended advice. However, in this case, the following estimated interval based advice for decision maker d_k could be produced

$$P_a = P_k + \frac{b_{k,\min}}{D_{kc}}(P_c - P_k) \tag{11}$$

with the adjustment direction being the same as described above in Eq. (10). Indeed, in this case, it would be $D_{ka} = b_{k,\min} \le \varepsilon_k$, which indicates the advice P_a is an acceptable advice for the decision maker d_k. In addition, it is also true that $p_{ij}^a + p_{ji}^a = 1$ and

$p_{ij}^a \in [\min(p_{ij}^k, p_{ij}^c), \max(p_{ij}^k, p_{ij}^c)] \subset [0,1]$, which implies that the adjusted preference $\overline{p}_{ij}^k$ will be closer to the collective preference p_{ij}^c than the preference it will replace and, consequently, it will lead to an improvement of the consensus level of the identified decision maker d_k.

(b) When $b_{k,\max} - b_{k,\min} > \theta$ and $D_{kc} \geq b_{k,\max}$, the estimated interval is not accurate and the identified decision maker d_k will not be willing to accept the collective preference relation P_c as a recommendation. For this reason, a bounded confidence β, which satisfies $b_{k,\min} < \beta < b_{k,\max}$, is estimated for identified decision maker d_k, so that the following estimated bounded confidence based advice can be formulated

$$P_a = P_k + \frac{\beta}{D_{kc}}(P_c - P_k) \tag{12}$$

with same adjustment direction as per Eq. (10). By doing this, it will be $D_{ka} = \beta \in (b_{k,\min}, b_{k,\max})$. If $\beta \leq \varepsilon_k$, decision maker d_k will accept the estimated bounded confidence based advice; otherwise, he/she will reject it. Furthermore, because of $D_{kc} \geq b_{k,\max} > \beta$, it is $p_{ij}^a + p_{ji}^a = 1$ and $p_{ij}^a \in [\min(p_{ij}^k, p_{ij}^c), \max(p_{ij}^k, p_{ij}^c)] \subset [0,1]$, and the consensus level of the identified decision maker d_k will be improved if he/she accepts the advice.

Due to the inaccurate estimated interval and the unacceptable of the collective preference, the advice is produced based on the estimated bounded confidence β with a twofold purpose: (i) to make the advice acceptable, and (ii) to better estimate the unknown bounded confidence in the next round. The purpose (ii) is presented in detail in the Learning algorithm 1 provided below.

(c) Under the following two situations (i) $b_{k,\max} - b_{k,\min} \leq \theta$ and $D_{kc} \leq b_{k,\min}$, and (ii) $b_{k,\max} - b_{k,\min} > \theta$ and $D_{kc} < b_{k,\max}$, the collective preference relation P_c will be provided to decision maker d_k as the advice, i.e., $P_a = P_c$. Meanwhile, the adjustment direction will be described by the Eq. (4). Although the advice is the same in both situations, their supported arguments are different. On the one hand, in situation (i), the estimate interval is accurate enough and the collective preference relation P_c is acceptable for the identified decision maker and it will improve his/her consensus level. On the other hand, in situation (ii), the collective preference relation P_c will also be accepted if $D_{kc} \leq b_{k,\min}$, and it would be a reasonable and helpful advice to further estimate the unknown bounded confidence of decision maker d_k if $b_{k,\min} < D_{kc} < b_{k,\max}$.

In order to improve the willingness of decision makers to accept the feedback mechanism advice, i.e., to provide more acceptable feedback mechanism advice, a learning algorithm is proposed to estimate the unknown bounded confidences of decision makers based on the preference adjustment, i.e., to obtain more accurate estimated intervals. In the following, the learning algorithm is formally presented as Learning algorithm 1.

Learning algorithm 1

Input: The preference P_k of the identified decision maker d_k, the advice P_a provided to the identified decision maker d_k, the estimated interval $[b_{k,\min}, b_{k,\max}]$ of decision maker d_k, and the bounded confidence threshold θ.

Output: The updated estimated interval $[\overline{b}_{k,\min}, \overline{b}_{k,\max}]$.

Step 1. Compute the distance D_{ka} between P_k and P_a using Eq. (7), i.e., $D_{ka} = \sum_{i=1}^{n-1}\sum_{j=i+1}^{n} | p_{ij}^k - p_{ij}^a | \big/ (n \cdot (n-1)/2)$. If $b_{k,\min} \leq D_{ka} \leq b_{k,\max}$, go to the next step; otherwise, let $[\overline{b}_{k,\min}, \overline{b}_{k,\max}] = [b_{k,\min}, b_{k,\max}]$, and go to Step 3.

Step 2. If decision maker d_k accepts the advice P_a and adjusts his/her preference, let $[\overline{b}_{k,\min}, \overline{b}_{k,\max}] = [D_{ka}, b_{k,\max}]$; otherwise, let $[\overline{b}_{k,\min}, \overline{b}_{k,\max}] = [b_{k,\max}, D_{ka}]$.

Step 3. Output the updated estimated interval $[\overline{b}_{k,\min}, \overline{b}_{k,\max}]$.

Notably, if $b_{k,\max} - b_{k,\min} \leq \theta$, the learning algorithm will not be activated to further estimate the bounded confidence of decision maker d_k. Then, the personalized feedback mechanism will produce advice based on the value $b_{k,\min}$ for decision maker d_k, because $b_{k,\min}$ play a similar role to the bounded confidence ε_k and the advice with $D_{ka} = b_{k,\min}$ is acceptable for him/her.

C. Algorithm for consensus reaching model with bounded confidence

According to the above description, the consensus reaching model with bounded confidence is presented below as Algorithm 2.

Algorithm 2: Consensus reaching model with bounded confidence

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Input: The initial preferences $\{P_1, P_2, \cdots, P_m\}$, the bounded confidence threshold θ, the weights for decision makers $\{w_1, w_2, \cdots, w_m\}$, the weights associated to the alternatives $\{\sigma_1, \sigma_2, \cdots, \sigma_n\}$, and the consensus threshold μ.

Output: The ranking of alternatives.

Step 1: Let $[b_{k,\min}, b_{k,\max}]$ be the estimated bounded confidence interval of decision maker d_k with unknown bounded confidence, and set $[b_{k,\min}, b_{k,\max}] = [0,1]$. Let $t = 0$, $P_{k,t} = (p_{ij}^{k,t})_{n \times n} = (p_{ij}^k)_{n \times n} (k = 1, 2, \cdots, m)$, and $[b_{k,\min}^t, b_{k,\max}^t] = [b_{k,\min}, b_{k,\max}]$.

Step 2: Compute the collective preference $P_{c,t} = (p_{ij}^{c,t})_{n \times n}$ at round t by aggregating the preferences $\{P_{1,t}, P_{2,t}, \cdots, P_{m,t}\}$ using the WA operator, i.e., $p_{ij}^{c,t} = \sum_{k=1}^{m} w_k p_{ij}^{k,t}$.

Step 3: Based on Eq. (3), calculate the group consensus level $cl_t = \sum_{k=1}^{m} w_k cl_t(d_k) \big/ m$, where $cl_t(d_k) = \sum_{i=1}^{n-1}\sum_{j=i+1}^{n} (1 - | p_{ij}^{k,t} - p_{ij}^{c,t} |) \big/ (n \cdot (n-1)/2)$. If $cl_t < \mu$, proceed to the next step; otherwise go to Step 6.

Step 4: Identify the decision maker d_k whose $cl_t(d_k)$ is the minimum. Let ε_k be his/her bounded confidence. Generate an estimated bounded confidence β_k^t from the interval $(b_{k,\min}^t, b_{k,\max}^t)$. Then, two cases are considered:

(1) If the bounded confidence ε_k is known, we advise that

$$\begin{cases} p_{ij}^{k,t+1} \in [\min(p_{ij}^{k,t}, p_{ij}^{c,t}), \max(p_{ij}^{k,t}, p_{ij}^{c,t})] & \text{if } D_{kc}^t \le \varepsilon_k \\ p_{ij}^{k,t+1} \in [\min(p_{ij}^{k,t}, p_{ij}^{a1,t}), \max(p_{ij}^{k,t}, p_{ij}^{a1,t})] & \text{if } D_{kc}^t > \varepsilon_k \end{cases} \text{ for } i \ge j \text{, and}$$

$p_{ij}^{k,t+1} = 1 - p_{ji}^{k,t+1}$ for $i < j$, where $p_{ij}^{a1,t} = p_{ij}^{k,t} + \frac{\varepsilon_k}{D_{kc}^t}(p_{ij}^{c,t} - p_{ij}^{k,t})$ and

$D_{kc}^t = \sum_{i=1}^{n-1} \sum_{j=i+1}^{n} | p_{ij}^{k,t} - p_{ij}^{c,t} | \big/ (n \cdot (n-1)/2)$;

(2) If the bounded confidence ε_k is unknown, we advise that

$$\begin{cases} p_{ij}^{k,t+1} \in [\min(p_{ij}^{k,t}, p_{ij}^{a2,t}), \max(p_{ij}^{k,t}, p_{ij}^{a2,t})] & \text{if } b_{k,\max}^t - b_{k,\min}^t \le \theta \text{ and } D_{kc}^t > b_{k,\min}^t \\ p_{ij}^{k,t+1} \in [\min(p_{ij}^{k,t}, p_{ij}^{a3,t}), \max(p_{ij}^{k,t}, p_{ij}^{a3,t})] & \text{if } b_{k,\max}^t - b_{k,\min}^t > \theta \text{ and } D_{kc}^t \ge b_{k,\max}^t \\ p_{ij}^{k,t+1} \in [\min(p_{ij}^{k,t}, p_{ij}^{c,t}), \max(p_{ij}^{k,t}, p_{ij}^{c,t})] & \text{if } b_{k,\max}^t - b_{k,\min}^t \le \theta \text{ and } D_{kc}^t \le b_{k,\min}^t \\ & \text{or } b_{k,\max}^t - b_{k,\min}^t > \theta \text{ and } D_{kc}^t < b_{k,\max}^t \end{cases}$$

for $i \ge j$, and $p_{ij}^{k,t+1} = 1 - p_{ji}^{k,t+1}$ for $i < j$, where

$p_{ij}^{a2,t} = p_{ij}^{k,t} + \frac{b_{k,\min}^t}{D_{kc}^t}(p_{ij}^{c,t} - p_{ij}^{k,t})$ and $p_{ij}^{a3,t} = p_{ij}^{k,t} + \frac{\beta_k^t}{D_{kc}^t}(p_{ij}^{c,t} - p_{ij}^{k,t})$.

Step 5: Based on the Learning algorithm 1, if $b_{k,\max}^t - b_{k,\min}^t > \theta$, let

$$\begin{cases} [b_{k,\min}^{t+1}, b_{k,\max}^{t+1}] = [\beta_k^t, b_{k,\max}^t] & \text{if } D_{kc}^t \ge b_{k,\max}^t \text{ and } \beta_k^t \le \varepsilon_k \\ [b_{k,\min}^{t+1}, b_{k,\max}^{t+1}] = [b_{k,\min}^t, \beta_k^t] & \text{if } D_{kc}^t \ge b_{k,\max}^t \text{ and } \beta_k^t > \varepsilon_k \\ [b_{k,\min}^{t+1}, b_{k,\max}^{t+1}] = [D_{kc}^t, b_{k,\max}^t] & \text{if } D_{kc}^t < b_{k,\max}^t \text{ and } D_{kc}^t \le \varepsilon_k \\ [b_{k,\min}^{t+1}, b_{k,\max}^{t+1}] = [b_{k,\min}^t, D_{kc}^t] & \text{if } D_{kc}^t < b_{k,\max}^t \text{ and } D_{kc}^t > \varepsilon_k \end{cases} \text{; otherwise, let}$$

$[b_{k,\min}^{t+1}, b_{k,\max}^{t+1}] = [b_{k,\min}^t, b_{k,\max}^t]$. Let $t = t+1$, then go to Step 2.

Step 6: Compute the evaluation values $Q_i = \sum_{j=1}^{n} \sigma_j \cdot p_{ij}^c$ from $P_{c,t}$ to obtain the ranking of alternatives. Output the ranking of alternatives.

IV. NUMERICAL AND SIMULATION ANALYSIS

A numerical analysis that illustrates the use of the proposed CRP with bounded confidence is presented, as well as a simulation analysis that discusses the effectiveness of the proposed model on consensus reaching.

A. Numerical example

A small GDM problem involving 6 decision makers, $\{d_1, d_2, \cdots, d_6\}$ equally important, and 4 alternatives, $\{x_1, x_2, \cdots, x_4\}$ is used to illustrate the new CRP with bounded confidence. Let $\{\varepsilon_1, \varepsilon_2, \cdots, \varepsilon_6\}$ be the bounded confidences associated to the decision makers. In this example, we assume that the bounded confidence ε_6 is unknown, and others' are known to be $\varepsilon_1 = 0.15$, $\varepsilon_2 = 0.25$, $\varepsilon_3 = 0.28$, $\varepsilon_4 = 0.12$, and $\varepsilon_5 = 0.2$. The consensus threshold μ is set to 0.85, whilst the bounded confidence threshold θ is set to 0.04. The initial preference relations provided by the decision makers are the following:

$$P_1 = \begin{pmatrix} 0.50 & 0.24 & 0.22 & 0.37 \\ 0.76 & 0.50 & 0.74 & 0.19 \\ 0.78 & 0.26 & 0.50 & 0.43 \\ 0.63 & 0.81 & 0.57 & 0.50 \end{pmatrix}, \quad P_1 = \begin{pmatrix} 0.50 & 0.25 & 0.28 & 0.20 \\ 0.75 & 0.50 & 0.82 & 0.74 \\ 0.72 & 0.18 & 0.50 & 0.76 \\ 0.80 & 0.26 & 0.24 & 0.50 \end{pmatrix},$$

$$P_3 = \begin{pmatrix} 0.50 & 0.46 & 0.92 & 0.21 \\ 0.54 & 0.50 & 0.83 & 0.78 \\ 0.08 & 0.17 & 0.50 & 0.13 \\ 0.79 & 0.22 & 0.87 & 0.50 \end{pmatrix}, \quad P_4 = \begin{pmatrix} 0.50 & 0.16 & 0.9 & 0.36 \\ 0.84 & 0.50 & 0.72 & 0.51 \\ 0.10 & 0.28 & 0.50 & 0.09 \\ 0.64 & 0.49 & 0.91 & 0.50 \end{pmatrix},$$

$$P_5 = \begin{pmatrix} 0.50 & 0.63 & 0.46 & 0.42 \\ 0.37 & 0.50 & 0.30 & 0.09 \\ 0.54 & 0.7 & 0.50 & 0.23 \\ 0.58 & 0.91 & 0.77 & 0.50 \end{pmatrix}, \quad P_6 = \begin{pmatrix} 0.50 & 0.77 & 0.12 & 0.81 \\ 0.23 & 0.50 & 0.34 & 0.80 \\ 0.88 & 0.66 & 0.50 & 0.71 \\ 0.19 & 0.20 & 0.29 & 0.50 \end{pmatrix}.$$

(1) CRP

(a) First round. Based on Eq. (1), the collective preference relation $P_{c,1}$ is obtained using the same equal weight 1/6 for each of the decision makers,

$$P_{c,1} = \begin{pmatrix} 0.5 & 0.418 & 0.483 & 0.395 \\ 0.582 & 0.5 & 0.625 & 0.518 \\ 0.517 & 0.375 & 0.5 & 0.392 \\ 0.605 & 0.482 & 0.608 & 0.5 \end{pmatrix}.$$

By Eq. (2), the decision makers' consensus levels are: $cl_1(d_1) = 0.842$, $cl_1(d_2) = 0.775$, $cl_1(d_3) = 0.768$, $cl_1(d_4) = 0.814$, $cl_1(d_5) = 0.804$, $cl_1(d_6) = 0.664$. By Eq. (3), the consensus level of the group is $cl_1 = 0.778$. Because $cl_1 = 0.778 < \mu = 0.85$, the decision makers need to adjust their preferences. According to the PIR, the decision maker with minimum consensus level is identified: d_6. According to the PDR, the collective preference relation $P_{c,1}$ is provided to decision maker d_6 based on the following three conditions: (a) his/her unknown bounded confidence, (b) $D_{6c,1} = 0.336 < b_{6,\max}^1 = 1$, and (c) $b_{6,\max}^1 - b_{6,\min}^1 = 1 > 0.04$.

(b) Second round. The decision maker d_6 declines the collective preference relation $P_{c,2}$ as the recommended advice, because it is far from his/her bounded confidence: $D_{6c,1} = 0.336 > \varepsilon_6$. Then, the $b_{6,\max}^2$ is estimated to be 0.336 according to the Learning algorithm 1. Due to the unchanged consensus level of each decision maker, the decision maker d_6 will still be advised to adjust his/her preference. According to the PDR, the new advice $P_{a,2}$, which satisfies $D_{6a,2} = 0.31$, is provided:

$$P_{a,2} = \begin{pmatrix} 0.5 & 0.435 & 0.466 & 0.415 \\ 0.565 & 0.5 & 0.612 & 0.532 \\ 0.534 & 0.388 & 0.5 & 0.407 \\ 0.585 & 0.468 & 0.593 & 0.5 \end{pmatrix}.$$

(c) Third round. Then, the adjusted preference of decision maker d_6 is obtained:

$$P_{6,3} = \begin{pmatrix} 0.5 & 0.479 & 0.421 & 0.466 \\ 0.522 & 0.5 & 0.576 & 0.567 \\ 0.579 & 0.424 & 0.5 & 0.446 \\ 0.534 & 0.434 & 0.554 & 0.5 \end{pmatrix}.$$

According to the learning algorithm, the $b_{6,\min}^3$ is estimated to be 0.31. Thus, it is $b_{6,\max}^3 - b_{6,\min}^3 = 0.026 < \theta = 0.04$ and the bounded confidence estimation of decision maker d_6 is accurate enough. Applying Eqs. (1) and (2), the consensus level of each decision maker is obtained: $cl_3(d_1) = 0.846$, $cl_3(d_2) = 0.777$, $cl_3(d_3) = 0.785$, $cl_3(d_4) = 0.843$, $cl_3(d_5) = 0.785$, and $cl_3(d_6) = 0.896$. Using Eq. (3), the new group consensus level becomes $cl_3 = 0.822$, which is still below the threshold value of consensus. At this round, decision maker d_2 is identified for preference adjustment. According to the PDR, the collective preference relation $P_{c,3}$ is used as new advice $P_{a,3}$,

$$P_{a,3} = \begin{pmatrix} 0.5 & 0.37 & 0.534 & 0.338 \\ 0.63 & 0.5 & 0.664 & 0.479 \\ 0.466 & 0.336 & 0.5 & 0.348 \\ 0.662 & 0.521 & 0.652 & 0.5 \end{pmatrix}.$$

(b) Fourth round. And the adjusted preference of decision maker d_2 is obtained:

$$P_{2,4} = \begin{pmatrix} 0.5 & 0.354 & 0.501 & 0.32 \\ 0.646 & 0.5 & 0.685 & 0.513 \\ 0.499 & 0.315 & 0.5 & 0.401 \\ 0.68 & 0.487 & 0.599 & 0.5 \end{pmatrix}.$$

Using Eqs. (1), (2) and (3), the group consensus level reaches the threshold value, $cl_4 = 0.85$; consequently, the CRP stops, and the selection process is activated.

(2) Selection process.

The temporal collective preference at the last round of the CRP becomes the final collective preference:

$$P_c = \begin{pmatrix} 0.5 & 0.387 & 0.57 & 0.358 \\ 0.613 & 0.5 & 0.642 & 0.442 \\ 0.43 & 0.358 & 0.5 & 0.288 \\ 0.642 & 0.558 & 0.712 & 0.5 \end{pmatrix}.$$

Assuming all alternatives are equally important, which translates in setting all weights in Eq. (5) to 1/4, the following ranking of alternatives is obtained: $x_4 > x_2 > x_1 > x_3$.

B. Simulation analysis

A simulation analysis that explores the effectiveness of the proposed new personalized feedback mechanism based consensus reaching model (abbreviated as the PFMCR model) on consensus reaching is presented, together with its comparison with the general consensus reaching model (abbreviated as the GCR model) of Section 2.1. In this simulation analysis the bounded confidences $\{\varepsilon_1, \varepsilon_2, \cdots, \varepsilon_m\}$ are randomly and uniformly generated within the interval $[\delta_{\min}, \delta_{\max}]$, and they are assumed to be partially known. Here, the number of unknown bounded confidences is denoted as parameter z. Without changing the essence of CRP, Eq. (12) is used to replace Eq. (9) in order to implement preference adjustment as an automatic process in this simulation analysis:

$$\begin{cases} \overline{p}_{ij}^k = \alpha_k \cdot p_{ij}^k + (1-\alpha_k) \cdot p_{ij}^a, & i \geq j \\ \overline{p}_{ij}^k = 1 - \overline{p}_{ji}^k, & i < j \end{cases} \tag{12}$$

where $\alpha_k \in [0,1]$ is a parameter to control the degree of advice.

Simulation analysis

Input: The number of decision makers m, the number of alternatives n, the maximum number of rounds T, the bounded confidence threshold θ, the number of unknown bounded confidences z, and the interval $[\delta_{\min}, \delta_{\max}]$.

Output: The consensus level $cl_t (t = 0,1,\cdots,T)$ at round t.

Step 1: Let $[b_{k,\min}, b_{k,\max}]$ be the estimated bounded confidence interval of decision maker d_k with unknown bounded confidence, and set $[b_{k,\min}, b_{k,\max}] = [0,1]$. Uniformly and randomly generate the bounded confidence ε_k from interval $[\delta_{\min}, \delta_{\max}]$ and parameter α_k from interval $[0,1]$. Generate $P_k = (p_{ij}^k)_{n\times n}$, where p_{ij}^k is uniformly and randomly from interval $[0,1]$ for $i \geq j$, $p_{ii}^k = 0.5$, and $p_{ij}^k = 1 - p_{ji}^k$ for $i < j$.

Step 2: Let $t = 0$, $P_{k,t} = (p_{ij}^{k,t})_{n\times n} = (p_{ij}^k)_{n\times n}$, and $[b_{k,\min}^t, b_{k,\max}^t] = [b_{k,\min}, b_{k,\max}]$.

Step 3: Based on Eq. (1), compute the collective preference $P_{c,t} = (p_{ij}^{c,t})_{n\times n}$ at round t by aggregating the preferences $\{P_{1,t}, P_{2,t}, \cdots, P_{m,t}\}$ with the weights $w_k = 1/m$ $(k = 1,2,\cdots,m)$, i.e., $p_{ij}^{c,t} = \sum_{k=1}^m p_{ij}^{k,t} \big/ m$.

Step 4: Based on Eqs. (2) and (3), compute the consensus level $cl_t = \sum_{k=1}^m cl_t(d_k) \big/ m$, where $cl_t(d_k) = \sum_{i=1}^{n-1} \sum_{j=i+1}^n (1 - |p_{ij}^{k,t} - p_{ij}^{c,t}|) \big/ (n\cdot(n-1)/2)$. If $t < T$, proceed to the next step; otherwise go to Step 7.

Step 5: Identify the decision maker d_k whose $cl_t(d_k)$ is the minimum. Uniformly and randomly generate an estimated bounded confidence β_k^t from the interval $(b_{k,\min}^t, b_{k,\max}^t)$. Then,

(1) If the bounded confidence ε_k is known, obtain $p_{ij}^{k,t+1}$ as follows

$$\begin{cases} p_{ij}^{k,t+1} = \alpha_k \cdot p_{ij}^{k,t} + (1-\alpha_k) \cdot p_{ij}^{c,t} & \text{if } D_{kc}^t \leq \varepsilon_k \\ p_{ij}^{k,t+1} = \alpha_k \cdot p_{ij}^{k,t} + (1-\alpha_k) \cdot p_{ij}^{a1,t} & \text{if } D_{kc}^t > \varepsilon_k \end{cases} \text{ for } i \geq j \text{, and}$$

$p_{ij}^{k,t+1} = 1 - p_{ji}^{k,t+1}$ for $i < j$, where $p_{ij}^{a1,t} = p_{ij}^{k,t} + \dfrac{\varepsilon_k}{D_{kc}^t}(p_{ij}^{c,t} - p_{ij}^{k,t})$ and $D_{kc}^t = \sum_{i=1}^{n-1} \sum_{j=i+1}^n |p_{ij}^{k,t} - p_{ij}^{c,t}| \big/ (n\cdot(n-1)/2)$;

(2) If the bounded confidence ε_k is unknown, obtain $p_{ij}^{k,t+1}$ as follows

(a) $\begin{cases} p_{ij}^{k,t+1} = \alpha_k \cdot p_{ij}^{k,t} + (1-\alpha_k) \cdot p_{ij}^{a2,t} & \text{if } b_{k,\max}^t - b_{k,\min}^t \leq \theta \text{ and } D_{kc}^t > b_{k,\min}^t \\ p_{ij}^{k,t+1} = \alpha_k \cdot p_{ij}^{k,t} + (1-\alpha_k) \cdot p_{ij}^{c,t} & \text{if } b_{k,\max}^t - b_{k,\min}^t \leq \theta \text{ and } D_{kc}^t \leq b_{k,\min}^t \end{cases}$

for $i \geq j$, and $p_{ij}^{k,t+1} = 1 - p_{ji}^{k,t+1}$ for $i < j$, where

$p_{ij}^{a2,t} = p_{ij}^{k,t} + \dfrac{b_{k,\min}^t}{D_{kc}^t}(p_{ij}^{c,t} - p_{ij}^{k,t})$;

(b) If $b_{k,\max}^t - b_{k,\min}^t > \theta$ and $D_{kc}^t \geq b_{k,\max}^t$, then

$$\begin{cases} p_{ij}^{k,t+1} = \alpha_k \times p_{ij}^{k,t} + (1-\alpha_k) \times p_{ij}^{a3,t} & \text{if } \beta_k^t \leq \varepsilon_k \\ p_{ij}^{k,t+1} = p_{ij}^{k,t} & \text{if } \beta_k^t > \varepsilon_k \end{cases} \text{ for } i \geq j \text{, and}$$

$p_{ij}^{k,t+1} = 1 - p_{ji}^{k,t+1}$ for $i < j$, where $p_{ij}^{a3,t} = p_{ij}^{k,t} + \dfrac{\beta_k^t}{D_{kc}^t}(p_{ij}^{c,t} - p_{ij}^{k,t})$;

(c) If $b_{k,\max}^t - b_{k,\min}^t > \theta$ and $D_{kc}^t < b_{k,\max}^t$, then

$$\begin{cases} p_{ij}^{k,t+1} = \alpha_k \cdot p_{ij}^{k,t} + (1-\alpha_k) \cdot p_{ij}^{c,t} & \text{if } \mathrm{D}_{kc}^{t} \le \varepsilon_k \\ p_{ij}^{k,t+1} = p_{ij}^{k,t} & \text{if } \mathrm{D}_{kc}^{t} > \varepsilon_k \end{cases} \text{ for } i \ge j \text{, and}$$

$p_{ij}^{k,t+1} = 1 - p_{ji}^{k,t+1}$ for $i < j$.

Step 6: Based on Learning algorithm 1, if $b_{k,\max}^{t} - b_{k,\min}^{t} > \theta$, let

$$\begin{cases} [b_{k,\min}^{t+1}, b_{k,\max}^{t+1}] = [\beta_k^t, b_{k,\max}^{t}] & \text{if } D_{kc}^{t} \ge b_{k,\max}^{t} \text{ and } \beta_k^t \le \varepsilon_k \\ [b_{k,\min}^{t+1}, b_{k,\max}^{t+1}] = [b_{k,\min}^{t}, \beta_k^t] & \text{if } D_{kc}^{t} \ge b_{k,\max}^{t} \text{ and } \beta_k^t > \varepsilon_k \\ [b_{k,\min}^{t+1}, b_{k,\max}^{t+1}] = [D_{kc}^{t}, b_{k,\max}^{t}] & \text{if } D_{kc}^{t} < b_{k,\max}^{t} \text{ and } D_{kc}^{t} \le \varepsilon_k \\ [b_{k,\min}^{t+1}, b_{k,\max}^{t+1}] = [b_{k,\min}^{t}, D_{kc}^{t}] & \text{if } D_{kc}^{t} < b_{k,\max}^{t} \text{ and } D_{kc}^{t} > \varepsilon_k \end{cases} \text{; otherwise, let}$$

$[b_{k,\min}^{t+1}, b_{k,\max}^{t+1}] = [b_{k,\min}^{t}, b_{k,\max}^{t}]$. Let $t = t+1$, and go to Step 3.

Step 7: Output the consensus level $cl_t (t = 0,1,\cdots,T)$ at each round.

Under the condition of considering the psychological factor of bounded confidence, we replace the Steps 5 and 6 in Simulation analysis with Steps 5' and 6' to obtain a modified version of Simulation analysis, which is used to investigate the effectiveness of the GCR model on consensus reaching. Steps 5' and 6' is presented below.

Step 5': Identify the decision maker d_k whose $cl_t(d_k)$ is the minimum. Then, obtain the adjusted preference $p_{ij}^{k,t+1}$ as follows

$$\begin{cases} p_{ij}^{k,t+1} = \alpha_k \cdot p_{ij}^{k,t} + (1-\alpha_k) \cdot p_{ij}^{c,t} & \text{if } D_{kc}^{t} \le \varepsilon_k \\ p_{ij}^{k,t+1} = p_{ij}^{k,t} & \text{if } D_{kc}^{t} > \varepsilon_k \end{cases} \text{ for } i \ge j \text{, and } p_{ij}^{k,t+1} = 1 - p_{ji}^{k,t+1}$$

for $i < j$, where $D_{kc}^{t} = \sum_{i=1}^{n-1} \sum_{j=i+1}^{n} | p_{ij}^{k,t} - p_{ij}^{c,t} | \big/ (n \cdot (n-1)/2)$.

Step 6': Let $t = t+1$, then go to Step 3.

To conduct the above simulation analysis, the necessary parameters are set as follows:

(1) Number of decision makers: $m = 6$.

(2) Number of alternatives: $n = 4$.

(3) Consensus threshold: $\mu = 1$.

(4) Maximum number of rounds: $T = 100$.

When setting different values for $[\delta_{\min}, \delta_{\max}]$, θ and z, we conduct this simulation analysis 1000 times to compute the average values for $cl_t (t = 0,1,\cdots,T)$, which are presented in Figs. 5-8.

From Figs. 5-8, the following observations are drawn:

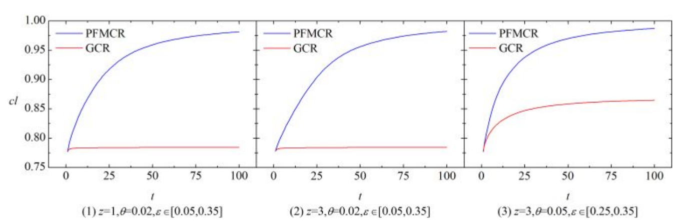

Fig. 5. Average cl_t values at each round in PFMCR and GCR models under different parameters z, θ, and $[\delta_{\min}, \delta_{\max}]$

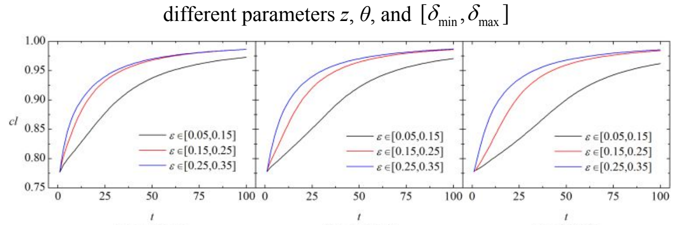

Fig. 6. Average cl_t values at each round in simulation analysis under different parameter $[\delta_{\min}, \delta_{\max}]$

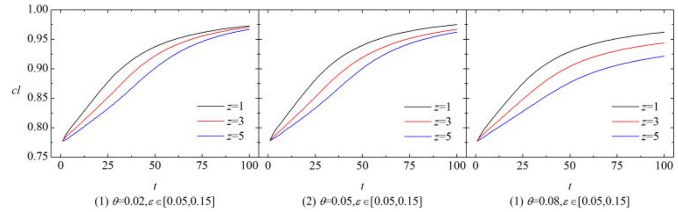

Fig. 7. Average cl_t values at each round in simulation analysis under different parameter z

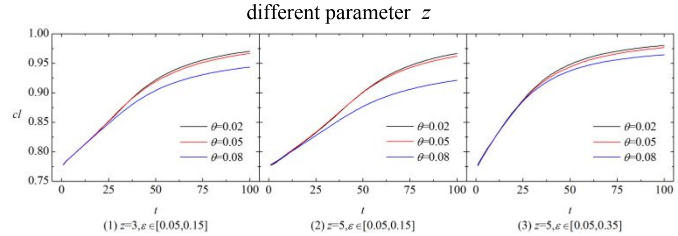

Fig. 8. Average cl_t values at each round in simulation analysis under different parameter θ

(1) The values obtained for cl_t with proposed model PFMCR are obviously greater than those obtained with the GCR model. Thus, the proposed CRP with bounded confidence model converges to consensus more rapidly than the GCR model.

(2) The values for cl_t increase as the value of $[\delta_{\min}, \delta_{\max}]$ increases, which means that the speed of convergence to consensus will faster when decision makers are more receptive to the advice, as it is obviously expected.

(3) Increasing of the value of z has the effect of decreasing the values of cl_t, i.e., group consensus is achieved more quickly by reducing the number of decision makers with unknown bounded confidences.

(4) Decreasing the value of θ increases the values of cl_t. This shows an accurate estimation of the unknown bounded confidences will result in the provision of acceptable advice, which in turn will facilitate the group reaching of consensus.

From these observations, it is concluded that the proposed personalized feedback mechanism based on bounded confidence can support decision makers improving the group consensus level with acceptable advices.

V. Conclusion

This paper proposes a novel consensus reaching model with personalized feedback mechanism that takes into account the individual willingness to accept feedback adjustment advices. The personalized feedback mechanism based consensus reaching model is driven by the use of bounded confidence model in opinion dynamics. In some situations, the bounded confidences of decision makers are unknown, and a learning algorithm is presented to address this issue. The detailed simulation and comparison experiments presented in this paper clearly show the effectiveness of the proposal. This is mainly due to the personalized feedback mechanism based consensus reaching model to effectively improve the individual willingness to accept feedback adjustment advice.

To support a large number of decision makers to reach consensus becomes necessary in GDM because of the development of information technology [33], [34], [44], [49], [52]. In the future, we plan to study the application of the findings of the present research to support consensus building in large-scale GDM.

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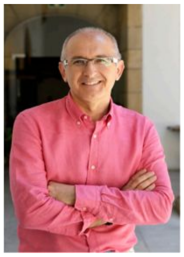

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