Multi-attribute Grey Target Decision-making Based on “Kernel” and Double Degree of Greyness

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Abstract

According to the characteristics of three-parameter interval grey number and the advantages of grey target, a multi-attribute grey target decision-making method is built. First, the “kernel” of the three-parameter interval grey number based on the most probability is defined, and the upper bound degree of greyness and the lower bound degree of greyness are separately defined for the asymmetry on the two sides, then the distance measure formula affected by the risk attitude of the decision maker is given. Considering the proximity of schemes to the optimal vector and the worst vector, the comprehensive off-target distances and their space projection on the line connecting the point of the positive bull’s eye and the negative bull’s eye are obtained, and the ranking of the schemes is ultimately determined. Finally, an example validates the rationality and effectiveness of the method, which may provide a new way of thinking in terms of research on grey decision-making theory and application.

Keywords: Three-parameter Interval Grey Number; “Kernel”; Double Degree of Greyness; Multi-attribute Grey Target Decision-making

1. Introduction

Multi-attribute decision-making has a wide range of applications in the fields of economy, management, engineering and military1-4, etc. However, due to the complexity of the socio-economic system and the limitations of people's cognitive abilities, decision makers can only evaluate the attributes and the index weights with interval numbers more than the exact values5. The upper limits of the range represent the most optimistic view and the lower limits represent the most conservative attitude. The corresponding decision problem is also somewhat uncertain, which makes it impossible for decision makers to follow the deterministic decisions. Therefore, it is of theoretical significance and practical value to make researches on uncertainty decision.

Grey target decision-making has been an important method in solving

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multi-attribute uncertain decision-making issues since it was proposed by professor Deng [6]. Liu & Lin defined off-target distance based on Euclidean distance and constructed a grey target decision model of $s$ dimension spherical [7]. Song et al. expanded the initial spherical grey target model to ellipsoid grey target model with positive and negative bull’s-eye [8]. Wang et al. improved the traditional off-target distance by using weighted Euclidean distance to eliminate index correlation and dimension difference [9]. Zeng & Liu established a twice-weighted grey target decision model based on interval grey numbers [10]. Zeng et al. established a grey target decision model by comparing the areas (cobweb) which are surrounded by the connections between each index value of the index set and the bull’s-eye [11]. Hu defined a new normalizing formula based on range transformation and put forward a grey target decision-making model [12]. Guo et al. constructed an annular grey target decision model based on different density of the decision making information [13]. Mao et al. proposed a multi-scale extended grey target decision method to solve the problem that the value distribution information of the interval grey numbers is asymmetrical [14].

In the previous studies, the decision-making information was indicated by traditional two-parameter interval grey numbers, not considering the most likely value of the interval grey number, and the results were not accurate enough. Luo defined a concept of three-parameter interval grey number, which highlighted the possibility value and made up for the poor information of grey number, and presented a multi-attribute decision-making method with three-parameter interval grey numbers [15]. Wang & Liu set up a ranking method based on the definition of relative superiority degree between three-parameter interval grey numbers and real numbers [16]. Wang & Liu discussed the properties of three-parameter interval grey numbers and established a grey target decision model [17]. Song proposed three-kinds of grey target decision-making models with three-parameter interval grey numbers [18]. Li et al. established a multi-attribute grey target decision method with three-parameter interval grey number [19]. Yan et al. studied dynamic grey target decision-making model with three-parameter grey numbers to establish the target distance reflecting both existing state and change trend [20].

However, some existing literature used three end points to study three-parameter interval grey numbers, which cannot represent the essential meaning of the grey numbers. Some scholars defined the “kernel” of three-parameter interval grey number by using weighted average of the three end points, which did not reflect the essence of the “center of gravity” and couldn’t characterize the special feature of the three-parameter interval grey number. From the definition of the three-parameter interval grey number, it can be seen that the center gravity of the three-parameter interval grey number is not necessarily at the midpoint. It may also have a certain bias to lower bound or upper bound, and the most likely value can be represented as nuclear of the interval grey number. Therefore, in this article, the “kernel” of the three-parameter interval grey number is redefined in the simplest form, and the lower bound degree of greyness and the upper bound degree of greyness of the three-parameter interval grey number are defined correspondingly. Considering the risk attitude of decision maker, the distance measure and ranking method of three-parameter interval grey numbers are given. On this basis, a multi-attribute decision-making method based on the “kernel” and the double degree of greyness of the three-parameter interval grey numbers is built. Finally, an application example is illustrated to verify the effectiveness and feasibility of the method.
2. Basic Concepts and Definitions

Definition 1[7] A three-parameter interval grey number is an interval grey number in which the value of the most likely gravity center is known and can be expressed as $\emptyset \in [a, \bar{a}, \overline{\alpha}]$, $(a < \bar{a} < \overline{\alpha})$. Among them, $a$ is lower bound of $\emptyset$, $\bar{a}$ is upper bound of $\emptyset$, $\overline{\alpha}$ is the most likely value of the interval grey number, called the center of gravity. When $\overline{\alpha}$ is unknown, the three-parameter interval grey number is the common interval grey number.

Definition 2 Let the background or domain of the three-parameter interval grey number $\emptyset$ be $\Omega$, $\mu^l(\emptyset) = \bar{a} - a$ be the lower measure of $\emptyset$, $\mu^u(\emptyset) = \overline{\alpha} - a$ be the total measure of $\emptyset$. In the absence of other distribution of $\emptyset$, $\hat{\emptyset} = \bar{a}$ is called as the “kernel” of $\emptyset$, $g^l(\emptyset) = \mu^l(\emptyset) / \mu(\Omega)$ is called as the lower degree of greyyness of $\emptyset$, $g^u(\emptyset) = \mu^u(\emptyset) / \mu(\Omega)$ is called as the upper degree of greyyness of $\emptyset$. A simplified form of the three-parameter interval grey number based on the “kernel” and the double degree of greyyness is $(\hat{\emptyset}, g^l(\emptyset), g^u(\emptyset))$.

Proposition 1[7] For a three-parameter interval grey number, its simplified form contains the information of upper bound, lower bound and the center gravity of the interval grey number, that is to say, the simplified form of the three-parameter interval grey number has same amount of information with its original form. In fact, when a three-parameter interval grey number $(\hat{\emptyset}, g^l(\emptyset), g^u(\emptyset))$ is given, its simplified form can be calculated according to the definition 2. In turn, when the simplified form is known, the “kernel”, the lower degree of greyyness and the upper degree of greyyness can be obtained, and the grey number is determined. There is a one-to-one correspondence between the interval grey number and its simplified form.

Definition 3 Let the domain of a three-parameter interval grey number be $\Omega$, when $\Omega \in [0, 1]$, the corresponding three-parameter interval grey number is called as a standard three-parameter interval grey number.

Proposition 2 Let $\emptyset$ be a standard three-parameter interval grey number, then $g^l(\emptyset) = \mu^l(\emptyset), g^u(\emptyset) = \mu^u(\emptyset), \text{and} g(\emptyset) = \mu(\emptyset)$.

Assume $\hat{\emptyset}_1(g_1^l, g_1^u)$ and $\hat{\emptyset}_2(g_2^l, g_2^u)$ are two three-parameter interval grey numbers, then the algorithm is as follows.

Rule 1 $\hat{\emptyset}_1(g_1^l, g_1^u) + \hat{\emptyset}_2(g_2^l, g_2^u) = (\hat{\emptyset}_1 + \hat{\emptyset}_2)(g_1^l + g_2^l, g_1^u + g_2^u)$

Rule 2 $\hat{\emptyset}_1(g_1^l, g_1^u) - \hat{\emptyset}_2(g_2^l, g_2^u) = (\hat{\emptyset}_1 - \hat{\emptyset}_2)(g_1^l + g_2^l, g_1^u + g_2^u)$

Rule 3 $\hat{\emptyset}_1(g_1^l, g_1^u) \times \hat{\emptyset}_2(g_2^l, g_2^u) = (\hat{\emptyset}_1 \times \hat{\emptyset}_2)(g_1^l + g_2^l, g_1^u + g_2^u)$

Rule 4 If $\hat{\emptyset}_2 \neq 0$, then $\hat{\emptyset}_1(g_1^l, g_1^u) / \hat{\emptyset}_2(g_2^l, g_2^u) = (\hat{\emptyset}_1 / \hat{\emptyset}_2)(g_1^l + g_2^l, g_1^u + g_2^u)$
Where \( \lambda = \frac{\hat{\Theta}}{\sum_{i=1}^{3} \hat{\Theta}_i} \) (\( i = 1, 2 \)) is the weight of the three-parameter interval grey number \( \hat{\Theta}_i \).

The algorithm can be extended to finite interval grey numbers to add, subtract, multiply and divide, etc.

Considering the particularity of the “center of gravity” in the three-parameter interval grey number and the influence of value range on decision-making under uncertainty, the distance between two three-parameter interval grey numbers is defined based on different risk attitude of the decision maker.

**Definition 4** Let \( \hat{\Theta}_1 \in [a_1^L, a_1^M, a_1^U] \) (\( a_1^L < a_1^M < a_1^U \)) and \( \hat{\Theta}_2 \in [a_2^L, a_2^M, a_2^U] \) (\( a_2^L < a_2^M < a_2^U \)) be two standard three-parameter interval grey numbers, the distance between them is defined as

\[
d(\hat{\Theta}_1, \hat{\Theta}_2) = \frac{1}{\lambda_1} \left[ \bar{a}_1 - \bar{a}_2 \right] + \lambda_2 \left[ g^x(\hat{\Theta}_1) - g^x(\hat{\Theta}_2) \right] + (1 - \lambda) \left[ g^y(\hat{\Theta}_1) - g^y(\hat{\Theta}_2) \right]
\]

(1)

Among them, \( \lambda \in [0,1] \) is risk factor which indicates the risk attitude of the decision maker. When \( \lambda > 0.5 \), it indicates that the decision maker is risk averse and tends to measure the distance between the two three-parameter grey intervals with the smaller endpoint; When \( \lambda < 0.5 \), it indicates that the decision maker is risk lover and tends to measure the distance with the larger endpoint; When \( \lambda = 0.5 \), it indicates that the decision maker is risk neutral and considered that the smaller endpoint and larger endpoint are equal important.

**Definition 5** Let \( \hat{\Theta}_1 \) and \( \hat{\Theta}_2 \) be two standard three-parameter interval grey numbers, then

1. If \( \hat{\Theta}_1 > \hat{\Theta}_2 \), then \( \hat{\Theta}_1 > \hat{\Theta}_2 \);
2. If \( \hat{\Theta}_1 < \hat{\Theta}_2 \), then \( \hat{\Theta}_1 < \hat{\Theta}_2 \);
3. If \( \hat{\Theta}_1 = \hat{\Theta}_2 \), then
   1. If \( g^x(\hat{\Theta}_1) - g^x(\hat{\Theta}_2) > g^x(\hat{\Theta}_2) - g^x(\hat{\Theta}_1) \), then \( \hat{\Theta}_1 > \hat{\Theta}_2 \);
   2. If \( g^x(\hat{\Theta}_1) - g^x(\hat{\Theta}_2) < g^x(\hat{\Theta}_2) - g^x(\hat{\Theta}_1) \), then \( \hat{\Theta}_1 < \hat{\Theta}_2 \);
   3. If \( g^x(\hat{\Theta}_1) - g^x(\hat{\Theta}_2) = g^x(\hat{\Theta}_2) - g^x(\hat{\Theta}_1) \), then
      1. If \( g(\hat{\Theta}_1) > g(\hat{\Theta}_2) \), then \( \hat{\Theta}_1 > \hat{\Theta}_2 \);
      2. If \( g(\hat{\Theta}_1) < g(\hat{\Theta}_2) \), then \( \hat{\Theta}_1 < \hat{\Theta}_2 \);
      3. If \( g(\hat{\Theta}_1) = g(\hat{\Theta}_2) \), then \( \hat{\Theta}_1 = \hat{\Theta}_2 \).

Where \( (\sim) \) has the same meaning with \( (> (<)) \) which is used to distinguish the comparison between three-parameter interval grey numbers from the comparison between real numbers, according to literatures [21, 22]. When both the lower degree of greyness and upper degree of greyness of the three-parameter interval grey numbers are zero, the comparison between the interval grey numbers is converted into the comparison between real numbers.
3. Decision-making Model Based on “Kernel” and Double Degree of Greyness

Let \( S = (s_1, s_2, \cdots, s_m) \) be a scheme set and \( C = \{c_1, c_2, \cdots, c_n\} \) be an index factor set of multi-attribute decision-making. Let \( \omega = (\omega_1, \omega_2, \cdots, \omega_n) \) be an attribute weights and \( \sum_{j=1}^{n} \omega_j = 1 \). The value of scheme \( s_i \) in index \( c_j \) evaluated by the decision maker is three-parameter interval grey number \( x_{ij}(\ominus)(i = 1, 2, \cdots, m; j = 1, 2, \cdots, n) \), then the evaluation matrix is

\[
X = \begin{bmatrix}
    x_{i1}(\ominus) & x_{i2}(\ominus) & \cdots & x_{in}(\ominus) \\
    \vdots & \vdots & \ddots & \vdots \\
    x_{m1}(\ominus) & x_{m2}(\ominus) & \cdots & x_{mn}(\ominus)
\end{bmatrix}
= \begin{bmatrix}
    [x_{11}^L, x_{11}^M, x_{11}^U] & [x_{12}^L, x_{12}^M, x_{12}^U] & \cdots & [x_{i1}^L, x_{11}^M, x_{11}^U] \\
    [x_{12}^L, x_{22}^M, x_{22}^U] & [x_{22}^L, x_{22}^M, x_{22}^U] & \cdots & [x_{i2}^L, x_{22}^M, x_{22}^U] \\
    \vdots & \vdots & \ddots & \vdots \\
    [x_{m1}^L, x_{m1}^M, x_{m1}^U] & [x_{m2}^L, x_{m2}^M, x_{m2}^U] & \cdots & [x_{mn}^L, x_{mn}^M, x_{mn}^U]
\end{bmatrix}
\]

In order to eliminate the influence of dimension, the evaluation matrix is normalized as follows \([15]\).

For efficiency indicators

\[
r_{ij}^L = \frac{x_{ij}^L - m}{M - m}, r_{ij}^M = \frac{x_{ij}^M - M - m}{M - m}, r_{ij}^U = \frac{x_{ij}^U - m}{M - m}
\]  

For cost indicators

\[
r_{ij}^L = \frac{M - x_{ij}^U}{M - m}, r_{ij}^M = \frac{M - x_{ij}^M}{M - m}, r_{ij}^U = \frac{M - x_{ij}^L}{M - m}
\]

Among them, \( M = \max_{1 \leq i \leq m}(x_{ij}^U) \), \( m = \min_{1 \leq i \leq m}(x_{ij}^L) \).

Then the normalized decision matrix is

\[
R = \begin{bmatrix}
    [r_{11}^L, r_{11}^M, r_{11}^U] & [r_{12}^L, r_{12}^M, r_{12}^U] & \cdots & [r_{i1}^L, r_{11}^M, r_{11}^U] \\
    [r_{12}^L, r_{22}^M, r_{22}^U] & [r_{22}^L, r_{22}^M, r_{22}^U] & \cdots & [r_{i2}^L, r_{22}^M, r_{22}^U] \\
    \vdots & \vdots & \ddots & \vdots \\
    [r_{m1}^L, r_{m1}^M, r_{m1}^U] & [r_{m2}^L, r_{m2}^M, r_{m2}^U] & \cdots & [r_{mn}^L, r_{mn}^M, r_{mn}^U]
\end{bmatrix}
\]

It can be seen that all the elements in the normalized decision matrix are standard three-parameter interval grey numbers. Then the simplified form of the decision-making matrix is

\[
R = \begin{bmatrix}
    r_{11}(\ominus) & r_{12}(\ominus) & \cdots & r_{1n}(\ominus) \\
    r_{21}(\ominus) & r_{22}(\ominus) & \cdots & r_{2n}(\ominus) \\
    \vdots & \vdots & \ddots & \vdots \\
    r_{m1}(\ominus) & r_{m2}(\ominus) & \cdots & r_{mn}(\ominus)
\end{bmatrix}
\]
Definition 6 Set \( r^+_j(\otimes) = \max_{1 \leq i \leq n}(r^+_i(\otimes)), \) (j = 1, 2, …, n), then
\[
r^+_j(\otimes) = [r^+_1(\otimes), r^+_2(\otimes), …, r^+_n(\otimes)]
\]
is referred to as the optimal vector of the grey target decision-making, which is also known as positive bull's eye.

Definition 7 Set \( r^-_j(\otimes) = \min_{1 \leq i \leq n}(r^-_i(\otimes)), \) (j = 1, 2, …, n), then
\[
r^-_j(\otimes) = [r^-_1(\otimes), r^-_2(\otimes), …, r^-_n(\otimes)]
\]
is referred to as the worst vector of the grey target decision-making, which is also known as negative bull's eye.

Definition 8 Set \( z^+_j = d(r^+_j(\otimes), r^+_1(\otimes)), \) then
\[
Z^+ = \begin{bmatrix}
z^+_1 & z^+_2 & \cdots & z^+_n \\
z^+_2 & z^+_2 & \cdots & z^+_2 \\
\vdots & \vdots & \ddots & \vdots \\
z^+_m & z^+_m & \cdots & z^+_m 
\end{bmatrix}
\]
is referred to as the coefficient matrix of the positive bull's eye, then the off-target distance between scheme \( s_i \) and the positive bull's eye is \( \eta^+_i = Z^+ \omega^T \).

Definition 9 Set \( z^-_j = d(r^-_j(\otimes), r^-_1(\otimes)), \) then
\[
Z^- = \begin{bmatrix}
z^-_1 & z^-_2 & \cdots & z^-_n \\
z^-_2 & z^-_2 & \cdots & z^-_2 \\
\vdots & \vdots & \ddots & \vdots \\
z^-_m & z^-_m & \cdots & z^-_m 
\end{bmatrix}
\]
is referred to as the coefficient matrix of the negative bull's eye, then the off-target distance between scheme \( s_i \) and the negative bull's eye is \( \eta^-_i = Z^- \omega^T \).

Definition 10 Set \( z^+_j = d(r^+_j(\otimes), r^-_1(\otimes)), \) then
\[
Z^0 = [z^+_1, z^+_2, \cdots, z^+_n]
\]
is referred to as the coefficient matrix between the optimal vector and the worst vector, then the distance between the optimal vector and the worst vector is \( \eta^0 = Z^0 \omega^T \).

The distance between the point of scheme \( s_i \) and the positive bull's eye satisfies \( \eta^+_i < \eta^0 \), the distance between the point of scheme \( s_i \) and the negative bull's eye satisfies \( \eta^-_i < \eta^0 \), so scheme \( s_i \), the positive bull's eye and the negative bull's eye are three-points that are either either lining in the straight line or forming a
Since both $\eta^+_i$ and $\eta^-_i$ are vectors, the projections of the off-target distance of scheme $s_i$ on the line connecting the positive bull's eye and the negative bull's eye are compared. By the cosine theorem, the projection of the off-target distance of scheme $s_i$ to the positive bull's eye is

$$
\eta^+_i = \eta^+_i \cos \theta = \frac{(\eta^+_i)^2 + (\eta^0)^2 - (\eta^-_i)^2}{2 \eta^0}
$$

Similarly, the projection of the off-target distance of scheme $s_i$ to the negative bull's eye is

$$
\eta^-_i = \eta^-_i \cos \theta = \frac{(\eta^-_i)^2 + (\eta^0)^2 - (\eta^+_i)^2}{2 \eta^0}
$$

So the comprehensive off-target distance of scheme $s_i$ is

$$
\eta_i = \eta^+_i - \eta^-_i = \frac{(\eta^+_i)^2 - (\eta^-_i)^2}{\eta^0}
$$

4. Decision-making Steps

From what have been discussed above, the general steps of the decision-making are shown below.

**Step 1** Construct the matrix of the evaluations based on the problem analysis.

**Step 2** Standardize the indicators by Eqs. (2) and (3), get a simplified form of the decision-making matrix based on “kernel” and the double degree of greyness.

**Step 3** Get the optimal point (positive bull's eye) and the worst point (negative bull's eye) by Eqs. (4) and (5), the coefficient matrix of positive bull's eye and the coefficient matrix of negative bull's eye by Eqs.(6) and (7), the coefficient matrix between the optimal vector and the worst vector by Eq. (8), respectively.

**Step 4** Obtain the projection distances and the comprehensive off-target distances by Eqs. (9), (10) and (11), respectively. Sort the schemes according to the comprehensive off-target distances.

5. Numerical Example

The example comes from reference [23]. It is assumed that the main parameters of influence carrier-borne machine are the following six items: maximum speed ($c_1$), freedom of the sea voyage ($c_2$), maximum payload ($c_3$), purchase cost ($c_4$), reliability ($c_5$), and flexibility ($c_6$). Now there exist four kinds of models for choice. Hence the factor set is $C = \{c_1, c_2, c_3, c_4, c_5, c_6\}$ and the scheme set is $S = (s_1, s_2, s_3, s_4)$. The normalized index weight vector given by experts is $\omega = (0.17, 0.12, 0.13, 0.13, 0.21, 0.24)$. The normalized evaluation matrix of three-parameter interval grey numbers is

$$
R = \begin{bmatrix}
[0.78, 0.80, 0.85] & [0.5, 0.55, 0.58] & [0.9, 0.95, 0.95] & [0.8, 0.82, 0.85] & [0.45, 0.5, 0.57] & [0.9, 0.95, 0.97] \\
[0.92, 0.95, 1.0] & [0.95, 0.97, 1.0] & [0.85, 0.86, 0.88] & [0.65, 0.69, 0.71] & [0.17, 0.20, 0.23] & [0.47, 0.51, 0.55] \\
[0.7, 0.72, 0.78] & [0.72, 0.74, 0.75] & [0.95, 0.98, 1.0] & [0.94, 0.97, 1.0] & [0.8, 0.83, 0.85] & [0.8, 0.82, 0.85] \\
[0.85, 0.88, 0.9] & [0.65, 0.67, 0.7] & [0.9, 0.95, 0.96] & [0.85, 0.9, 0.93] & [0.46, 0.5, 0.52] & [0.48, 0.5, 0.52]
\end{bmatrix}
$$
The simplified form of the decision-making matrix based on the “kernel” and the double degrees of greyness is

\[
R = \begin{bmatrix}
0.80_{(0.02,0.03)} & 0.55_{(0.05,0.01)} & 0.95_{(0.05,0.00)} & 0.82_{(0.02,0.03)} & 0.50_{(0.05,0.07)} & 0.95_{(0.05,0.02)} \\
0.95_{(0.03,0.05)} & 0.97_{(0.02,0.03)} & 0.86_{(0.01,0.02)} & 0.69_{(0.04,0.02)} & 0.20_{(0.03,0.03)} & 0.51_{(0.04,0.04)} \\
0.72_{(0.02,0.06)} & 0.74_{(0.02,0.01)} & 0.98_{(0.03,0.02)} & 0.97_{(0.03,0.03)} & 0.83_{(0.03,0.02)} & 0.82_{(0.02,0.03)} \\
0.88_{(0.03,0.02)} & 0.67_{(0.02,0.03)} & 0.95_{(0.05,0.01)} & 0.90_{(0.05,0.03)} & 0.50_{(0.04,0.02)} & 0.50_{(0.02,0.02)}
\end{bmatrix}
\]

The optimal vector of the decision-making is

\[
r^+ = [0.95_{(0.03,0.05)}, 0.97_{(0.02,0.03)}, 0.98_{(0.03,0.02)}, 0.97_{(0.03,0.03)}, 0.83_{(0.03,0.02)}, 0.95_{(0.05,0.02)}]
\]

The worst vector of the decision-making is

\[
r^- = [0.72_{(0.02,0.06)}, 0.55_{(0.05,0.00)}, 0.86_{(0.01,0.02)}, 0.69_{(0.04,0.02)}, 0.69_{(0.04,0.02)}, 0.50_{(0.02,0.02)}]
\]

Considering the risk attitude of the decision maker, when \( \varepsilon \) increases from 0 to 1, the off-target distances of scheme \( s_i \) to positive bull's eye and negative bull's eye are obtained, and the schemes are sorted according to the comprehensive target distances, as shown in table 1.

<table>
<thead>
<tr>
<th>( \varepsilon )</th>
<th>( \eta_1 )</th>
<th>0.1</th>
<th>0.2</th>
<th>0.3</th>
<th>0.4</th>
<th>0.5</th>
<th>0.6</th>
<th>0.7</th>
<th>0.8</th>
<th>0.9</th>
</tr>
</thead>
<tbody>
<tr>
<td>( s_2 )</td>
<td>-0.6586</td>
<td>-0.6549</td>
<td>-0.6511</td>
<td>-0.6474</td>
<td>-0.6436</td>
<td>-0.6397</td>
<td>-0.6358</td>
<td>-0.6319</td>
<td>-0.6280</td>
<td></td>
</tr>
<tr>
<td>( s_3 )</td>
<td>1.1857</td>
<td>1.1814</td>
<td>1.1770</td>
<td>1.1727</td>
<td>1.1684</td>
<td>1.1642</td>
<td>1.1599</td>
<td>1.1558</td>
<td>1.1516</td>
<td></td>
</tr>
<tr>
<td>( s_4 )</td>
<td>-0.7730</td>
<td>-0.7815</td>
<td>-0.7901</td>
<td>-0.7989</td>
<td>-0.8077</td>
<td>-0.8166</td>
<td>-0.8255</td>
<td>-0.8346</td>
<td>-0.8438</td>
<td></td>
</tr>
<tr>
<td>( s_1 )</td>
<td>0.9056</td>
<td>0.9152</td>
<td>0.9249</td>
<td>0.9346</td>
<td>0.9445</td>
<td>0.9544</td>
<td>0.9645</td>
<td>0.9746</td>
<td>0.9848</td>
<td></td>
</tr>
</tbody>
</table>

The optimal scheme is \( S_2 \), \( S_2 \), \( S_2 \), \( S_2 \), \( S_2 \), \( S_2 \), \( S_2 \), \( S_2 \), \( S_2 \), \( S_2 \).

It can be seen from table 1 that no matter what the value \( \varepsilon \) is, the sequence of the target distance of the schemes remains unchanged as \( \eta_2 > \eta_4 > \eta_1 > \eta_3 \), therefore schemes can be sorted as \( S_3 > S_1 > S_4 > S_2 \). The calculation is consistent not only with that of literature [23], but also with that of artificial judgment. It can also be seen that the off-target distance of scheme \( S_1 \) and \( S_4 \) are increased with the growth of \( \varepsilon \). This shows that these two schemes are negatively related with the risk attitude of the decision maker. However, along with the increase of \( \varepsilon \), the other two off-target distances of scheme \( S_2 \) and \( S_3 \) are decreased. This shows that they are positively related with the risk attitude of the decision maker, that it is to say, the more risk loving the decision maker is, the higher the comprehensive evaluations of the two schemes are.

Due to the considerable difference among the evaluation, the sequence of the scheme is unchanged with the risk factor. Compared with the decision-making model of literature [23], this method pays more attention to the important role of "center of gravity" and the double degree of greyness based on the decision-making information, and it can better reflect the intrinsic characteristics of the...
three-parameter interval grey number.

6. Conclusion

In the current study, combined with the merits of grey target, a multi-attribute grey target decision-making model under the information of three-parameter interval grey number is presented. By redefining the “kernel”, the upper degree of greyness, and the lower degree of greyness, the simplified form of the three-parameter interval grey number is given. A distance formula with the risk preference factor is proposed and a more practical ranking method is put forward too. On this basis, a three-parameter interval grey number decision-making model based on the “kernel” and the double degree of greyness is put forward. The method highlights the most likely value of the interval grey number and embodies the basic principle of the grey system. Compared with other models, it takes into account the characteristics of the three-parameter interval grey number and has a strong consistency with the existing grey multi-criteria decision-making methods. The results of the numerical example verified the scientificity and rationality of the model.

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