A Constrained Optimization Approach to Dynamic State Estimation for Power Systems including PMU Measurements

Liang Hu, Zidong Wang, Izaz Rahman and Xiaohui Liu

Abstract

In this paper, a hybrid filter algorithm is introduced, aiming at including the synchronized phasor measurement units (PMU) measurements into the dynamic state estimator designed for power systems with traditional measurements. Based on the dynamics of the power system and its traditional measurements, an extended Kalman filter (EKF) is designed. As data dropouts inevitably exist in the transmission channels of traditional measurements from the meters to the control centre, the missing measurement phenomenon has been taken into account in the state estimator design. Due to the intrinsic disadvantage in the technique of local linear approximation to nonlinearities, the EKF yields valid but low accurate state estimates. To improve the EKF estimation performance, in this paper, the PMU measurements are treated as inequality constraints of the states with the aid of the statistical criterion. In the framework of EKF, it becomes a constrained state estimation problem to incorporate the state constraints into the system states. Through the maximization probability method, this problem is converted into a constrained optimization problem, which is easily tackled by the Particle Swarm Optimization (PSO) algorithm together with the penalty function approach. The proposed algorithm is then employed to the power system state estimation. It is shown that the proposed algorithm gives much improved performances over the traditional EKF method.

Keywords

Extended Kalman filter, power system state estimation, missing measurements, particle swarm optimization, constrained optimization

I. INTRODUCTION

State estimation (SE) has long been one of the fundamental problems in power system. Traditional SE approach is typically static where the single-scan weighted least squares estimators are adopted [1]. Static SE method exhibits the features of fast convergence and easy implementation, but it suffers from the accuracy problems since the dynamics of the power system is ignored [2]. With rapid development of the sensing techniques, online monitoring has recently become popular which gives rise to the renewed research interests on the design of the dynamic state estimator (DSE), see e.g. [3–8] and the references therein. Comparing with the static state estimation scheme, the DSE is capable of achieving better estimation accuracy since more information about the state evolution is utilized. Another advantage of the DSE is its ability to provide prediction database that could be adopted as a set of pseudo-measurements in case of missing data or meter outages in the power grids. However, it is to be noted that, though these advantages have been mentioned in

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L. Hu, Z. Wang, I. Rahman and X. Liu are with the Department of Information Systems and Computing, Brunel University, Uxbridge, Middlesex, UB8 3PH, United Kingdom. (Email: Zidong.Wang@brunel.ac.uk)
almost all the literature on DSE, to the best of the authors’ knowledge, a detailed DSE scheme considering missing data has not yet been proposed, except the initiatory result reported in [9].

Traditional SE has been based on the measurements for power systems collected via remote terminal units (RTUs). Power service providers send the status and analog values of key generators and transmission lines to the control centre through communication networks. As discussed in [11], the communication constraints, such as limited bandwidth have inevitably led to network-induced phenomena such as random communication delays and missing measurements. Recently, the SE problem has been investigated in [12] for power systems with random measurement delays. As for missing measurements, a conventional way is to treat them as normal bad data without in-depth characterization of the dropouts. Very recently, the missing measurement problem has been tackled in [9] where a certain stochastic variable is involved in the estimator which renders the difficulties in the implementation.

Recently, more advanced synchronized phasor measurement technology are applied in power systems, which makes it possible to measure the system states in a more accurate and timely way. Comparing with traditional measurements, PMU measurements are more timely and accurate due to the intrinsic advantages of the PMU. PMU measurements can be as fast as 30 measurements/s, and all PMU measurements are synchronized by the global position system’s (GPS) universal clock [13]. Unfortunately, for economic reasons, it is not affordable to replace all the RTUs with PMUs in the foreseeable future [14,15]. In other words, only partial states could be measured directly by PMUs and the rest would have to be estimated by using the conventional RTUs. As such, an emerging research issue is how to integrate PMU measurements into traditional SE algorithms, and this issue has started to gain some initial research attention, see [14–17]. It should be noted that all the corresponding results available in the literature have been concerned with static SE problems, and the DSE problem in the presence of partial PMU involvements remains as a challenging topic of research [13]. This situation motivates our current investigation.

The main purpose of our proposed research is to solve the above challenging problems by making the first attempt to develop a constrained optimization approach to dynamic state estimation for power systems including PMU measurement. As to the missing RTU measurement issue, it will be brought into attention when designing the EKF for the power system. The phenomenon of missing measurements is assumed to occur in a random way and the missing probability for each channel is governed by an individual random variable satisfying a certain probability distribution over the interval [0,1]. We will look at how missing measurements impact on the overall estimation performance. That is, how tolerant our estimator would be against the probabilistic missing measurements.

As to incorporating the PMU measurements into the aforementioned EKF estimator, we convert the PMU measurements into a set of inequality constraints based on the well-known 3-sigma rule of Gaussian distribution. Since the EKF problem with state constraints is essentially a constrained optimization problem, we develop a particle swarming optimization (PSO) algorithm to solve the constrained optimization problem. The reason for choosing the PSO algorithm is that PSO makes few or no assumptions about the problem being optimized and can search very large spaces of candidate solutions, which is particularly suitable for the online nature of the SE problem for power systems. As PSO has been developed primarily as an unconstrained optimization method, the penalty function approach is utilized to convert the constrained optimization problem to a unconstrained optimization one. It is to be noted that the penalty function approach associated with PSO has been one of the most popular constraint-handling techniques because of its simplicity for converting
a constrained optimization problem into an unconstrained optimization one by adding a penalty term to the objective function [18,19].

In this paper, we aim to design a novel state estimation algorithm for power systems with unreliable underlying communication networks when partial measurements by PMUs are available. A hybrid EKF and PSO algorithm is developed to estimate the states of power system including PMU measurements. The main contribution of this paper is threefold: 1) A new dynamic state estimation scheme is first proposed to improve the estimation performance of power system including PMU measurements. Such a scheme has the advantages of being scalable to the numbers of the installed PMUs and of being easily incorporated with the existing DSE software; 2) Practical issues of missing measurements in communication network are investigated thoroughly and a modified EKF algorithm is developed which is insensitive to the measurement unreliability in terms of acceptable probability; 3) Extensive comparative experiments have been implemented based on different missing rates of the RTU measurements. From the results of these experiments, we can find that our proposed estimation algorithm provides better performance than the traditional EKF when missing measurements exist.

The reminder of this paper is organized as follows. In Section II, the dynamic model of the power systems is briefly introduced, and the PMU measurement is represented as a set of inequality constraints on systems states. In Section III, the EKF estimation problem with the inequality constraints on the states is converted to a constrained optimization problem by the maximum probability method. The PSO algorithm together with the penalty function approach for the constrained optimization problem is described in Section IV. The results of case studies performed on the 14-bus IEEE benchmark system are presented and analyzed in Section V. Finally, the paper is concluded in Section VI.

Notation The notation used here is fairly standard except where otherwise stated. $\mathbb{R}^n$ and $\mathbb{R}^{n \times m}$ denote, respectively, the $n$ dimensional Euclidean space and the set of all $n \times m$ real matrices. $I$ denotes the identity matrix of compatible dimension, and $I_{m,1}$ denotes the $m$ dimensional vector with all elements equal to 1. For given matrices $A$ and $B$ with the same dimension, $\circ$ is the Hadamard product with this product being defined as $[A \circ B]_{ij} = [A_{ij} \cdot B_{ij}]$. $A^T$ represents the transpose of $A$, and $\mathbb{E}\{x\}$ stands for the expectation of the stochastic variable $x$. $|C|$ describes the determinants of a square matrix $C$. $\text{diag}\{\cdots \}$ stands for a block-diagonal matrix and the notation $\text{diag}_n\{\ast\}$ is employed to stand for $\text{diag}\{\ast, \cdots, \ast\}$.

II. PROBLEM FORMULATION AND PRELIMINARIES

A. System Model with Missing RTU Measurements

In this paper, the following dynamic equation is used to model the power system containing $N$ buses:

$$x(k + 1) - u = A(x(k) - u) + \omega(k)$$

where the state $x(k) \in \mathbb{R}^n$ is the vector of phasor voltages of all buses, and $u \in \mathbb{R}^n$ is the nominal centre of the normal state. $\omega(k)$ is a Gaussian sequence with zero mean and covariance matrix $W(k)$. $A$ is a known matrix with appropriate dimensions and represents how fast the transitions between states are. The initial value of state $x(0)$ is a white Gaussian noise with mean value $\bar{x}(0)$ and covariance matrix $\Sigma(0|0)$. For a power system with $N$ buses, the state $x(k)$ can be chosen as $x(k) = [x_{r,1}(k) \ x_{r,2}(k) \ \cdots \ x_{r,N}(k) \ x_{i,1}(k) \ x_{i,2}(k) \ \cdots \ x_{i,N}(k)]^T$ where $x_{r,l}(k)$ and $x_{i,l}(k)$ represent the real and imaginary voltage of the $l$th bus, respectively.
For the purpose of simplicity, (1) can be represented in the following compact form:

\[ x(k + 1) = Ax(k) + Bu + \omega(k) \]

with \( B = I - A \), where \( B \) is associated with the trend behavior of the state trajectory.

The ideal measurement \( z^{(r)}(k) \in \mathbb{R}^m \) collected by RTUs is as follows

\[ z^{(r)}(k) = [V^T(k) \ P^T(k) \ Q^T(k) \ P^T_f(k) \ Q^T_f(k)]^T \]

where \( V^T(k) = [V_1(k) \ V_2(k) \cdots V_{n_v}(k)] \) denotes the bus voltage magnitude measurements, \( P^T(k) = [P_1(k) \ P_2(k) \cdots P_{n_p}(k)] \) and \( Q(k) = [Q_1(k) \ Q_2(k) \cdots Q_{n_q}(k)] \) stand for the real and reactive bus power injections measurements, and \( P_f(k) = [P_{f1}(k) \ P_{f2}(k) \cdots \ P_{f_{n_l}}(k)] \) and \( Q_f(k) = [Q_{f1}(k) \ Q_{f2}(k) \cdots \ Q_{f_{n_l}}(k)] \) are the real and reactive transmission line power flows. \( n_v, n_p \) and \( n_q \) are equal to the number of voltage meters, power meters installed at the buses and power flow meters installed at the lines. Assuming the general two-port \( \pi \)-model for the network branches, the explicit element for each aforementioned measurement is given as follows (for the purpose of simplicity, the time instant \( k \) is omitted):

\[
V_s = \sqrt{x_{r,s}^2 + x_{i,s}^2} \\
P_s = x_{r,s} \sum_{j \in \mathbb{N}_s} (G_{sj}x_{r,j} - B_{sj}x_{i,j}) + x_{i,s} \sum_{j \in \mathbb{N}_s} (G_{sj}x_{i,j} + B_{sj}x_{r,j}) \\
Q_s = x_{r,s} \sum_{j \in \mathbb{N}_s} (G_{sj}x_{r,j} - B_{sj}x_{i,j}) - x_{r,s} \sum_{j \in \mathbb{N}_s} (G_{sj}x_{i,j} + B_{sj}x_{r,j}) \\
P_{fs} := P_{ftj} = (x_{r,t}^2 + x_{i,t}^2)(g_{tj}^0 + g_{tj}) - x_{r,t}x_{r,j}g_{tj} - x_{i,t}x_{i,j}g_{tj} - x_{r,t}x_{i,j}b_{tj} - x_{i,t}x_{r,j}b_{tj} \\
Q_{fs} := Q_{ftj} = -(x_{r,t}^2 + x_{i,t}^2)(b_{tj}^0 + b_{tj}) - x_{i,t}x_{r,j}g_{tj} + x_{r,t}x_{i,j}g_{tj} + x_{r,t}x_{r,j}b_{tj} + x_{i,t}x_{i,j}b_{tj}
\]

where \( G_{sj} + jB_{sj} \) is the \( sj \)th element of the complex bus admittance matrix, \( g_{tj} + jb_{tj} \) is the admittance of the series branch connecting bus \( t \) and \( j \), \( g_{tj}^0 + jb_{tj}^0 \) is the half admittance of the shunt branch of the line collecting bus \( t \) and \( j \) in the \( \pi \)-model circuit, and \( \mathbb{N}_s \) is the set of bus numbers which are directly connected to bus \( s \). The above measurements are all accessible but not necessary. Generally, there are \( m \) measurements and \( n \) state variables, \( m > n \).

Taking the measurement noise into consideration, \( z^{(r)}(k) \) can be rewritten as the following compact form

\[ z^{(r)}(k) = h(x(k)) + v_1(k) \]

where the nonlinear function \( h(x(k)) \) is determined by (3), and \( v_1(k) \) is the RTU measurement noise, which is also a Gaussian noise with zero mean and covariance matrix \( R_1(k) \). Assume that \( \omega(k) \) and \( v_1(k) \) are uncorrelated with \( x(0) \) and with each other.

Considering missing measurements, the actual measurement \( z(k) \) is described by

\[ z(k) = \Xi(k)h(x(k)) + v_1(k) \]

where \( \Xi(k) = \text{diag}\{\gamma_1(k), \gamma_2(k), \cdots, \gamma_m(k)\} \) with \( \gamma_i(k) \) \((i = 1, 2, \cdots, m)\) being \( m \) unrelated random variables. \( \Xi(k) \) is also unrelated with \( \omega(k) \), \( v_1(k) \) and \( x(0) \). Furthermore, it is assumed that the stochastic variable \( \gamma_i(k) \) is a Bernoulli-distributed white noise sequence taking values on 0 or 1 with:

\[ \text{Prob}\{\gamma_i(k) = 0\} = 1 - \mu_i(k), \quad \text{Prob}\{\gamma_i(k) = 1\} = \mu_i(k) \]
where the value of \( \text{Prob}\{\gamma_i(k) = 0\} \) is also called the missing rate of the \( i \)th measurement.

In RTU measurements, one bus is usually chosen as the reference bus for all the other buses to obtain the relative phase angles, while in PMU measurements, all PMU measurements provide the direct phase angles with respect to the time reference provided by the GPS system. In this paper, we use both RTU and PMU measurements, then all the bus phase angles are relative to the reference dictated by the GPS [14], as a result, no reference buses are needed. Traditionally, the phasor angles and magnitudes are treated separately as state variables, whereas an alternative representation (i.e. the real and imaginary voltages of the buses) is adopted as state variables in this paper, just as in [20].

**B. PMU Measurements and Inequality Constraints**

Recently, PMUs have been increasingly deployed in power systems as PMUs are able to provide more accurate and more timely measurements than RTUs.

A PMU measures not only the voltage phasor of the bus where it is installed, but also the current measurements of the lines connecting to the bus. As theoretically PMU measurements are inherently in the rectangular form [21], it is suggested that PMUs should provide the data in both angular and rectangular form in the IEEE standard c37.118-2005 [22]. In this paper, both the state variables and measured variables are in the rectangular form, which makes a linear measurement model. Assume that the installed PMU buses are labeled \( s_1, s_2, \ldots, s_M \). The measurement \( z_{s_i}^{(p)} \in \mathbb{R}^{2(1+N_i)} \) obtained from the \( i \)th PMU deployed at the bus \( s_i \), can be described as follows,

\[
z_{s_i}^{(p)} = \begin{bmatrix} z_{r,s_i}^{(p)} \\ z_{i,s_i}^{(p)} \\ z_{r,i}^{(p)} \\ \cdots \\ z_{r,N_i}^{(p)} \\ z_{i,N_i}^{(p)} \end{bmatrix}^T.
\]

Specifically, the voltage measurement in the above vector is as follows,

\[
z_{r,s_i}^{(p)} = x_{r,s_i}, \quad z_{i,s_i}^{(p)} = x_{i,s_i}
\]

where \( z_{r,s_i}^{(p)} \) and \( z_{i,s_i}^{(p)} \) are the real and imaginary voltage measurements of bus \( s_i \), respectively.

The current measurement of the line connecting the bus \( s_i \) and \( t_i^{(s)} \) is as follows,

\[
\begin{aligned}
z_{r,i}^{(p)} &= (x_{r,s_i} - x_{r,t_i^{(s)}})g_{s_i,t_i^{(s)}} - (x_{i,s_i} - x_{i,t_i^{(s)}})b_{s_i,t_i^{(s)}} + x_{r,s_i}g_{s_i,t_i^{(s)}} - x_{i,s_i}b_{s_i,t_i^{(s)}} \\
z_{i,i}^{(p)} &= (x_{i,s_i} - x_{i,t_i^{(s)}})g_{s_i,t_i^{(s)}} + (x_{r,s_i} - x_{r,t_i^{(s)}})b_{s_i,t_i^{(s)}} + x_{i,s_i}g_{s_i,t_i^{(s)}} + x_{r,s_i}b_{s_i,t_i^{(s)}}
\end{aligned}
\]

where \( z_{r,s_i}^{(p)} \) and \( z_{i,s_i}^{(p)} \) are the real and imaginary current measurements, respectively.

Considering the measurement noise, the PMU measurements can be presented in the following compact vector form:

\[
z^{(p)}(k) = H^{(p)} x^{(p)}(k) + v_2(k)
\]

where \( z^{(p)} = [z_{s_1}^{(p)}, \ldots, z_{s_M}^{(p)}] \in \mathbb{R}^{mp} \) with \( mp = 2(M + N_1 + N_2 + \ldots + N_I) \). \( x^{(p)}(k) \in \mathbb{R}^{n_1} \) is the state vector related to the PMU measurements, and the set of all elements of which is a subset of the set of all elements of \( x(k) \), that is, \( x^{(p)}(k) = Cx(k), C \in \mathbb{R}^{ni \times n_1}, n_1 \leq n \). \( v_2(k) \) is the RTU measurement noise, which is also a Gaussian noise with zero mean and covariance matrix \( R_2(k) \). \( H^{(p)} \) can be obtained directly from the equation (7). It can be found that the measurement \( z^{(p)}(k) \) is linearly related to the state \( x^{(p)}(k) \).

It has been reported that the standard deviation of the errors of PMU measurements is one to two order magnitude less than the one of traditional RTU measurements in [14, 16, 23], which illustrates that PMU measurements are more accurate than RTU measurements.
measurements are much more accurate than traditional RTU measurements. As $R_2(k)$ is always a real symmetric matrix, we can find a transformation matrix $M(k)$ of appropriate dimension such that the matrix $M(k)R_2(k)M^T(k)$ is diagonal. Accordingly, we can obtain the following equation from (8):

$$M(k)z^{(p)}(k) = M(k)H^{[p]}x^P(k) + M(k)v_2(k)$$

(9)

where $M(k)v_2(k)$ is still a Gaussian noise with zero mean and covariance matrix $M(k)R_2(k)M^T(k)$. Based on the well known 3-sigma rule of Gaussian distribution, we can conclude that the following inequality sets are satisfied with probability 99.7%.

$$-3\tilde{R}_2(k) \leq M(k)z^{(p)}(k) - M(k)H^{[p]}x^P(k) \leq 3\tilde{R}_2(k)$$

(10)

where $\tilde{R}_2(k) \triangleq M(k)R_2(k)M^T(k)I_{m_1,1}$. From the perspective of engineering applications, it is reasonable to assume that the above inequality sets are satisfied all the time. So far, we have converted the PMU measurements into a set of inequality constraints of the states for the power system.

III. Filter Schemes for RTU and PMU Measurements

A. EKF Design for the System with RTU Measurements

In this subsection, we first introduce the EKF approach to estimate the system state for the system (2) with missing measurements (5). The EKF is of the following form:

$$\begin{align*}
\dot{x}(k|k-1) &= A\hat{x}(k-1|k-1) + Bu \\
\hat{x}(k|k) &= \hat{x}(k|k-1) + K(k)[z(k) - \Xi(k)h(\hat{x}(k|k-1))] \\
\end{align*}$$

(11)

(12)

where $\hat{x}(k|k)$ is the estimate of $x(k)$ at time instant $k$ with $\hat{x}(0|0) = \hat{x}(0)$, and $\hat{x}(k|k-1)$ is the one-step prediction of $x(k)$ at time $k-1$. $K(k)$ is the filter gain to be determined at time instant $k$, and $\Xi(k) := \mathbb{E}\{\Xi(k)\} = \text{diag}\{\mu_1(k), \mu_2(k), \ldots, \mu_m(k)\}$. Denote

$$H(k) = \left. \frac{\partial h(x(k))}{\partial x(k)} \right|_{x(k)=\hat{x}(k|k-1)}.$$ 

(13)

and then the filter gain $K(k)$ can be obtained by using the following recursive algorithm:

$$\begin{align*}
P(k|k-1) &= AP(k-1|k-1)A^T + W(k-1) \\
P(k|k) &= [I - K(k)\Xi(k)H(k)]P^{-1}(k|k-1) \\
S(k) &= \Xi(k)\circ (h(\hat{x}(k|k-1))h^T(\hat{x}(k|k-1))) + \Xi(k)\circ (H(k)P(k|k-1)H^T(k)) \\
&\quad + \Xi(k)H(k)P(k|k-1)H^T(k)\Xi(k) + R(k) \\
K(k) &= P(k|k-1)H^T(k)\Xi(k)S^{-1}(k) \\
\end{align*}$$

(14)

(15)

(16)

(17)

where $\Xi(k) := \text{diag}\{\tilde{\mu}_1(k), \tilde{\mu}_2(k), \ldots, \tilde{\mu}_m(k)\}$ with $\tilde{\mu}_i(k) = \mu_i(k)(1 - \mu_i(k))$ ($i = 1, 2, \ldots, m$), $P(k|k-1)$ and $P(k|k)$ are the one-step prediction error and filtering error covariance matrices defined as follows,

$$\begin{align*}
\hat{x}(k|k-1) &= x(k) - \hat{x}(k|k-1), \quad P(k|k-1) = \mathbb{E}\{(\hat{x}(k|k-1) - \hat{x}(k|k-1))^T\} \\
\hat{x}(k|k) &= x(k) - \hat{x}(k|k), \quad P(k|k) = \mathbb{E}\{(\hat{x}(k|k) - \hat{x}(k|k))^T\}. \\
\end{align*}$$
Remark 1: How to make use of both RTU and PMU measurements to obtain an optimal estimated state is a challenging issue today. Mainly speaking, two static estimation schemes are proposed in [14–17,23]. One is to process both kinds of measurements simultaneously after transforming the two kinds of measurements into a common coordination. The other one is a two-stage scheme: estimated states are obtained by employing RTU measurements and RTU measurements, respectively, and then such estimates are fused based on the estimation fusion formula. However, these schemes have not taken into account the dynamics of the power systems, nor are applicable to the system with randomly missing measurements.

B. The Maximum Probability Method

For the constrained estimation problem, it is difficult to incorporate the inequality/equality constraint of system states into traditional EKF estimator. However, the maximum probability method [24], [25] has been exploited successfully to convert the constrained estimation into a constrained optimization problem after each step of the EKF algorithm and, therefore, this method is chosen to handle the constrained EKF problem in this paper.

For the purpose of simplicity, the time instant $k$ is omitted in this subsection. It is known from [26] that, based on Kalman filter theory, the state estimate of $x$ maximizes the conditional probability density

$$ P(x|Z) = (2\pi)^{-n/2} |P|^{-\frac{1}{2}} \exp\{-\frac{1}{2}(x - \bar{x})^T P^{-1} (x - \bar{x})\}, \tag{18} $$

where $n$ is the dimension of $x$, $P$ is the covariance of the Kalman filter estimate, $Z$ denotes the column vector that contains the measurements $\{z(0), z(1), \ldots, z(k)\}$, and $\bar{x}$ is the conditional mean of $x$ given the measurement $Z$.

The constrained EKF can be derived by finding an estimate $\hat{x}$ such that the conditional probability $P(\hat{x}|Z)$ is maximized and $\hat{x}$ satisfies the constraint (10). Since maximizing $P(\hat{x}|Z)$ is equivalent to maximizing its natural logarithm, the problem to be solved can be expressed as

$$ \max \ln P(\hat{x}|Y) \Rightarrow \min (\hat{x} - \bar{x})^T P^{-1} (\hat{x} - \bar{x}) $$

such that $-3\bar{R}_2 \leq Mz^{(p)} - MH^{(p)}C\hat{x} \leq 3\bar{R}_2$. \tag{19}

So far, the constrained state estimation problem has been converted into an equivalent constrained optimization problem that can be solved after each time step of the EKF algorithm.

IV. Particle Swarm optimization for Constrained Optimization Problem

Particle Swarm optimization (PSO) is a metaheuristic that optimizes a problem by iteratively searching in a large spaces of candidate solutions [10]. In PSO, a population of candidate solutions (called as particles) moves in the search space according to two simple mathematic formulae over the particle’s position and velocity. More specifically, each particle’s movement is influenced by its local best known position and also the best known positions, which are updated by other particles, in the search space. By such an iterate approach, the swarm of the particles moves towards the best solutions. The velocity and position of the particle at the next iteration are updated according to the following equations:

$$ v_i(s + 1) = \omega v_i(s) + c_1 r_1(p_i(s) - x_i(s)) + c_2 r_2(p_g(s) - x_i(s)) \tag{20} $$

$$ x_i(s + 1) = x_i(s) + v_i(s + 1) $$
TABLE I

The nominal voltage at normal states

<table>
<thead>
<tr>
<th>Bus</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>7</th>
<th>8</th>
<th>9</th>
<th>10</th>
<th>11</th>
<th>12</th>
<th>13</th>
<th>14</th>
</tr>
</thead>
<tbody>
<tr>
<td>Real voltages</td>
<td>1.0600</td>
<td>1.0368</td>
<td>0.9609</td>
<td>0.9858</td>
<td>0.9958</td>
<td>1.0016</td>
<td>1.0022</td>
<td>1.0270</td>
<td>0.9827</td>
<td>0.9760</td>
<td>0.9850</td>
<td>0.9806</td>
<td>0.9755</td>
<td>0.9552</td>
</tr>
<tr>
<td>Imaginary voltages</td>
<td>0</td>
<td>0.0943</td>
<td>0.2173</td>
<td>0.1821</td>
<td>0.1565</td>
<td>0.2694</td>
<td>0.2512</td>
<td>0.2645</td>
<td>0.2743</td>
<td>0.2744</td>
<td>0.2724</td>
<td>0.2759</td>
<td>0.2748</td>
<td>0.2812</td>
</tr>
</tbody>
</table>

where \( x_i(s) = [x_{i1}(s), \ldots, x_{id}(s)] \), \( x_i(s) \) is the position of the \( i \)th particle at the \( s \)th iteration, and \( x_i(s) \in [x_{min,n}, x_{max,n}] \), with \( x_{min,n} \) and \( x_{max,n} \) being the lower and the upper bounds for all particles’ positions. \( v_i(s) = [v_{i1}(s), \ldots, v_{id}(s)] \), \( v_i(s) \) is the velocity of the \( i \)th particle at the \( s \)th iteration. \( \omega \) is the inertia weight, \( c_1 \) and \( c_1 \) are called acceleration coefficients, namely, cognitive and social parameters, respectively. \( r_1 \) and \( r_2 \) are two uniform random number samples from \([0, 1] \). \( p_i(s) \) is the local best position encountered by \( i \)th particle at the \( s \)th iteration, and \( p_g(s) \) is the global best position in the swarm at the \( s \)th iteration.

PSO has been successfully applied to optimization problems in a variety of areas. As to constrained optimization problem, PSO is still valid with the aid of the popular constraint-handling technique: the penalty function approach [18]. By using the penalty function approach, a constrained optimization problem can be converted into a corresponding unconstrained optimization one by adding a penalty term to the original objective function.

In this paper, the penalty function \( F(x) \) is defined as

\[
F(x) = f(x) + h(s)g(x), \quad x \in \mathbb{R}^n
\]  

(21)

where \( f(x) \) is the original objective function of the constrained optimization problem in (19), \( h(s) \) is a penalty factor defined as \( g(x) = \theta \sum_{i=1}^{m1} q_i^2(x) \). Here, \( \theta \) is a given scalar and \( q_i(x) = \max\{0, c_i(x)\} \) with \( c_i(x) = \frac{|M_i(k)x_k^p - H^{(p)}x^p|}{3R_{2i}(k)} - 1, \quad i = 1, \ldots, m_1 \), where \( M_i(k) \) and \( R_{2i}(k) \) are the \( i \)th row of \( M(k) \) and \( R_2(k) \) in inequality (10).

V. Simulation results

In this section, the proposed hybrid algorithm of EKF and PSO is tested in the case study of the IEEE 14-bus test system. The simulation is implemented in Matlab with the Matpower package [27]. First, the IEEE 14-bus test system can be model as (1) with parameters \( A = \text{diag}_{28}(0.98), \quad B = \text{diag}_{28}(0.02) \) and \( W(k) = \text{diag}_{28}(0.01^2) \). The nominal centre \( u \) of the normal state is the base-case voltages given in Table I. Furthermore, assume that the initial voltages of all buses are at flat start, that is, \( x_{r,l}(0) = 1 \) p.u, \( x_{i,l}(0) = 0 \) for all \( l = 1, 2, \ldots, 14 \).

The measurement configuration is the same as the one used in [14], in which RTU measurements consist of three categories: the voltage magnitude at bus 1, power injections at the bus 3, 5, 13 and 14, and power flows at branches 1-2, 1-5, 2-5, 3-4, 4-7, 4-9, 6-11, 6-12, 6-13, 7-8, 7-9, 9-10, 9-14, 10-11, 12-13 and 13-14. In addition, PMUs are deployed at buses 2, 7 and 9. Furthermore, the covariance matrices of the traditional RTU measurement noise and PMU measurement noise are \( R_1(k) = \text{diag}_{43}(0.1^2) \) and \( R_2(k) = \text{diag}_{28}(0.01^2) \), respectively.

In this test system, three comparative experiments regarding the estimation accuracy are carried out as follows:

1) Both the proposed EKF, which considers the missing measurements with certain missing rate, and the tra-
When the missing rate of the measurements varies from zero to higher values, the proposed EKF is implemented in all the cases; 3) The state estimations based on the proposed EKF with/without PSO algorithm are compared.

In order to provide more generalised case results and higher statistical significance, 100 Monte-Carlo simulations are run in the last two experiments. The Mean square errors (MSE) are adopted to evaluate the estimation accuracy, where \( \text{MSE}_i \) denotes the mean square error (MSE) for the estimate of the \( i \)th state, i.e.,

\[
\text{MSE}_i(k) = \frac{1}{100} \sum_{j=1}^{100} (x_i(k) - \hat{x}_i(k))^2.
\]

To evaluate the average estimation performance of all state variables, average mean square error (AMSE) is defined as

\[
\text{AMSE}(k) := \frac{1}{n} \sum_{i=1}^{n} \text{MSE}_i(k),
\]

where \( n \) is the number of the state variables. In all the figures, “R.V” and “I.V” denote the real voltage and the imaginary voltage, respectively.
A. Traditional EKF vs the Proposed EKF

In this case, the probability density function for the missing $\Xi(k)$ is as follows,

$$
\text{Prob}\{\Xi_i(k) = 0\} = 0.15, \quad \text{Prob}\{\Xi_i(k) = 1\} = 0.85.
$$

The expectation can be easily calculated as $\mu_i(k) = 0.85$. The estimated states of the representative buses 2, 5, 9, 14 obtained from traditional EKF, in which missing measurements have not been considered, are plotted in Fig. 1 and Fig. 2. While the counterparts obtained from the proposed EKF considering missing measurements, are plotted in Fig. 3 and Fig. 4. From the comparison, we can find that, our proposed EKF still performs well, while the traditional EKF state estimate cannot track the real states when missing measurements happen randomly.
Fig. 5. The MSEs of the estimated states for non-missing and missing measurements

Fig. 6. The MSEs of the estimated states for missing rates 0.02 and 0.05

TABLE II

<table>
<thead>
<tr>
<th>Time instant</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>7</th>
<th>8</th>
<th>9</th>
<th>10</th>
<th>11</th>
<th>12</th>
<th>13</th>
<th>14</th>
<th>15</th>
</tr>
</thead>
<tbody>
<tr>
<td>MR = 0.00 (EKF)</td>
<td>0.1350</td>
<td>0.1609</td>
<td>0.1748</td>
<td>0.2121</td>
<td>0.2342</td>
<td>0.2366</td>
<td>0.2564</td>
<td>0.2767</td>
<td>0.2651</td>
<td>0.2767</td>
<td>0.2651</td>
<td>0.2707</td>
<td>0.2601</td>
<td>0.2572</td>
<td>0.2503</td>
</tr>
<tr>
<td>MR = 0.02 (EKF)</td>
<td>0.1344</td>
<td>0.1784</td>
<td>0.2091</td>
<td>0.2134</td>
<td>0.2123</td>
<td>0.2356</td>
<td>0.2550</td>
<td>0.2425</td>
<td>0.2787</td>
<td>0.2884</td>
<td>0.2851</td>
<td>0.3136</td>
<td>0.3202</td>
<td>0.3253</td>
<td>0.3578</td>
</tr>
<tr>
<td>MR = 0.15 (EKF)</td>
<td>0.1416</td>
<td>0.1836</td>
<td>0.2152</td>
<td>0.2596</td>
<td>0.2673</td>
<td>0.2913</td>
<td>0.3341</td>
<td>0.3590</td>
<td>0.3813</td>
<td>0.3858</td>
<td>0.4410</td>
<td>0.4615</td>
<td>0.4801</td>
<td>0.4612</td>
<td></td>
</tr>
<tr>
<td>MR = 0.15 (Hybrid)</td>
<td>0.1342</td>
<td>0.1725</td>
<td>0.2066</td>
<td>0.2047</td>
<td>0.2226</td>
<td>0.2450</td>
<td>0.2682</td>
<td>0.2818</td>
<td>0.3135</td>
<td>0.2985</td>
<td>0.2958</td>
<td>0.2950</td>
<td>0.3050</td>
<td>0.3199</td>
<td>0.3049</td>
</tr>
</tbody>
</table>
Fig. 7. The estimated states by EKF and the proposed hybrid algorithm

B. EKF for Missing Measurements

As to different missing rates, three missing rates are considered: one with missing rate 0.15, another one with missing rate 0.02, and the last one without missing measurements.

The MSEs of the estimated states of buses 2, 9 for all the three cases are compared in Fig. 5 and Fig. 6. The AMSE\((k)\) in all three cases are given in the first three rows of Table II, for \(k = 1, 2, \ldots, 15\). In the Table II, “MR” stands for the missing rate, “EKF” stands for the proposed EKF algorithm and “Hybrid” stands for the proposed hybrid EKF and PSO algorithm. The results in the Table II are magnified \(10^3\) times for clearness. From the comparisons, we can find that the less the missing rate is, the more accurate the state estimation obtained from the proposed EKF algorithm is.

C. EKF vs Hybrid EKF and PSO Algorithm

In this case, it is assumed that the missing rate is 0.15. Regarding the penalty function parameters, we choose \(\theta = 1000\) and \(h(k) = 1\) in all the iteration steps. For the same test system, one realization of the EKF and one realization of the hybrid algorithm are simulated simultaneously, and the estimated real voltages of buses 2, 9 obtained from the two algorithms are illustrated in Fig. 7. It is seen that the trajectory by the proposed hybrid approach is much more closer to the true state trajectory than the one only by the EKF. The MSE2 and MSE9 at all time instants for both algorithms are plotted in Fig. 8. We can find that for the same state variable, the MSE of EKF-based state estimation is bigger than the MSE of the state estimation obtained from the hybrid algorithm. Especially, when the accumulated error of EKF-based state estimation becomes bigger after several integrations, the subsequent PSO algorithm can refine the state estimation and diminish the error. The AMSEs of EKF and of the proposed hybrid algorithm are given in the last two rows of Table II. It can be find the AMSE\(k\) of EKF is bigger than the one of the proposed hybrid algorithm at each time instant. From these comparisons, we can conclude that the constrained state estimation obtained from the hybrid EKF and PSO algorithm improves the unconstrained EKF state estimation.
VI. Conclusion

In this paper, we have developed a hybrid EKF and PSO algorithm for power system dynamic state estimation. In consideration of the missing traditional measurements, a novel EKF estimator is designed for the power system. To incorporate the PMU measurements in the designed EKF estimator, the 3-sigma rule in Statistics is employed to represent the PMU measurements as a set of inequality constraints. By using the maximization probability method, the constraints of system states can be handled by combining the EKF algorithm with a constrained optimization problem. Then, the PSO algorithm together with the penalty function is employed to solve the constrained optimization problem. Simulations illustrate the performance of the proposed algorithm under multiple conditions. Higher estimation accuracy is achieved with this algorithm, which verifies the effectiveness of the propose method.

References


