Confidence Based Consensus in Environments with High Uncertainty and Incomplete Information

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Abstract. With the incorporation of web 2.0 frameworks the complexity of decision making situations has exponentially increased, involving in many cases many experts, and a potentially huge number of different alternatives, leading the experts to present uncertainty with the preferences provided. In this context, intuitionistic fuzzy preference relations play a key role as they provide the experts with means to allocate the uncertainty inherent in their proposed opinions. However, in many occasions the experts are unable to give a preference due to different reasons, therefore effective mechanisms to cope with missing informations are more than necessary. In this contribution, we present a new group decision making (GDM) approach able to estimate the missing information and at the same time implements a mechanism to bring the experts' opinions closer in an iterative process in which the experts' confidence plays a key role.

Keywords. Group decision making, Uncertainty, Consensus, Intuitionistic Fuzzy Preference relations

1. Introduction

Group decision making (GDM) procedures provide a framework for experts to express their opinions and interact with the common goal of achieving agreement and selecting the best possible alternative. Obviously the experts could have different backgrounds and different points of view about the solution of the problem, or even they may be unable to propose an accurate solution due to the inherent uncertainty that involves many GDM processes. In these situations Intuitionistic fuzzy preference relations, IFPRs, based on Attanasov's Intuitionistc Fuzzy sets, [6], suppose an interesting framework for the experts to express their judgements, since they allow them to allocate certain levels of uncertainty in their opinions. This type of PR includes a membership degree, a non-

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membership degree and a hesitation index to model experts' subjective preferences. Recently, the use of IAPRs in decision making in uncertain environments has attracted the attention of many researchers and various group decision making approaches have been presented. [26, 29, 32].

Some of these applications suppose that the experts provide complete information about their preferences. However in real world situations the experts might not have a precise or sufficient level of knowledge of the problem. As a consequence, they do not provide all the information that is required [2, 9, 11, 20]. In the literature, we can find a wide variety of methods that deal with missing preference relations for well known types of PR such us fuzzy reciprocal PR or Linguistic Preference relations. Most of these approaches are based on the selection of an appropriate methodology to 'build' the matrix, and/or to assign importance values to experts based not on the amount of information provided but on how consistent the information provided is. Most of these methods estimate expert's missing values using just his/her own assessments and consistency criteria to avoid incompatibility, some examples can be found in [1, 3–5, 12, 14, 16, 17, 21, 22]. More concretely for the case of IFPR, in the literature we can find two iterative approaches that deals with incomplete information [30]. However in many occasions these approaches are not designed to deal with uncertainty or carries out complex transformation between different types of preference representations leading to a lost of precision. The first objective of this contribution is to present a completion approach based on the equivalence between the set of Intuitionistic Fuzzy Preference Relations and the set of asymmetric fuzzy Preference relations demonstrated in [28].

On the other hand, a key issue in GDM consists on achieving a full and unanimous agreement among all the experts. However in the majority of the occasions is not reachable in practice. An alternative approach is to use softer consensus measures [8] that better represent the human perception of the essence of consensus. These approaches define the consensus process as a dynamic and iterative group discussion coordinated by a moderator that helps experts to bring their opinions closer. To guide the consensus process different indicators has been used in the literature. Among them we can highlight twofold: Consistency and Similarity. Consistency is linked to rationality of individuals whereas similarity can be interpreted as a measure of general or widespread agreement. By combining both consistency and similarity functions, Herrera-Viedma et al. [16] developed a feedback mechanism to provide advice to experts in order to increase the consensus level of the group. Furthermore, Chiclana et al. in [10] designed a two stage model with a first stage aiming to reach acceptable consistency level while the second one was used to achieve a predefined consensus level. Focusing on the case of IFPRs there are already available some consensus models in the literature [30, 31].

However , in order to guide the consensus processes, in environments where the experts present high level of uncertainty in their opinions, other measures should be taken into account as well [28]. In this sense, It has been found that freely interacting groups choose the positions of their most confident members as their group decisions. This phenomenon has been witnessed with groups discussing a mathematical puzzle [19], a recall task [25] and a recognition task [18], concluding that confidence was a significant predictor of influence. Furthermore Guha et al. state in [15] that in any real field decision making situation when experts give their responses to a particular alternative, their confidence level regarding the opinions are very important. In this sense, in [28] it has been presented an approach which asses the experts degree of confidence directly from the experts opinions expressed by means of IFPRs, and so it allows to take into account this valuable information in the decision making process. Therefore, the second objective of this contribution is to present a new confidence-consistency based consensus model that takes into consideration both the experts' consistency and confidence levels to implement a feedback mechanism to support experts to change some of their preference values using simple advice rules that aim at increasing the level of agreement while, at the same time, keeping a high degree of consistency.

The rest of the paper is set out as follows: Section 2 presents the main mathematical frameworks for representing preferences and the basics concepts needed throughout the rest of the paper. Section 3.2 introduces the new Confidence-Consistency based consensus approach with completion of missing information. Finally, Section 4 draws conclusions and presents some future work.

2. Background

In group decision making problems, once the set of feasible alternatives (X) is identified, experts are called to express their opinions or preferences on such set. Different preference elicitation methods were compared in [23], where it was concluded that pairwise comparison methods are more accurate than non-pairwise methods because they allow the expert to focus on two alternatives at a time. A comparison of two alternatives by an expert can lead to the preference of one alternative to the other or to a state of indifference between them. Obviously, there is the possibility of an expert being unable to compare them. Two main mathematical models based on the concept of preference relation can be used in this context. In the first one, a preference relation is defined for each one of the above three possible preference states mentioned above (preference, indifference, incomparability) [13], which is usually referred to as a preference structure on the set of alternatives [24]. The second one integrates the three possible preference states into a single preference relation [7]. In this paper, we focus on the second one as per the following definition:

Definition 1 (Preference Relation)**.** *A preference relation P on a set X is a binary relation $\mu_P : X \times X \to D$, where D is the domain of representation of preference degrees provided by the decision maker.*

A preference relation P may be conveniently represented by a matrix $P = (p_{ij})$ of dimension $\#X$, with $p_{ij} = \mu_P(x_i, x_j)$ being interpreted as the degree or intensity of preference of alternative x_i over x_j. The elements of P can be of a numeric or linguistic nature, i.e., could represent numeric or linguistic preferences, respectively. The main types of numeric preference relations used in decision making are: crisp preference relations, additive preference relations, multiplicative preference relations, interval-valued preference relations and intuitionistic preference relations. A comprehensive survey of them have been reported on [33], which the reader is encouraged to consult for further particulars. In this contribution, the focus is on fuzzy preference relations and intuitionistic fuzzy preference relations.

2.1. Fuzzy Set and Fuzzy Preference Relation

Definition 2 (Fuzzy Set)**.** *Let U be a universal set defined in a specific problem, with a generic element denoted by x. A fuzzy set X in U is a set of ordered pairs:*

$$X = \{(x, \mu_X(x)) | x \in U\}$$

where $\mu_X : U \to [0,1]$ is called the membership function of A and $\mu_X(x)$ represents the degree of membership of the element x in X.

The degree of non-membership of the element x in X is here defined as $\nu_X(x) = 1 - \mu_X(x)$. Thus, $\mu_X(x) + \nu_X(x) = 1$.

Definition 3 (Fuzzy Preference Relation)**.** *A fuzzy preference relation $R = (r_{ij})$ on a finite set of alternatives X is a fuzzy relation in $X \times X$ that is characterised by a membership function $\mu_R : X \times X \longrightarrow [0,1]$ with the following interpretation:*

- *$r_{ij} = 1$ indicates the maximum degree of preference for x_i over x_j*
- *$r_{ij} \in]0.5, 1[$ indicates a definite preference for x_i over x_j*
- *$r_{ij} = 1/2$ indicates indifference between x_i and x_j*

When

$$r_{ij} + r_{ji} = 1 \ \forall i, j \in \{1, \ldots, n\}$$

is imposed the fuzzy preference relation is called reciprocal.

2.2. Intuitionistic Fuzzy Set and Intuitionistic Fuzzy Preference Relation

The concept of an *Intuitionistic Fuzzy Set* (IFS) was introduced by Atanassov in [6]:

Definition 4 (Intuitionistic Fuzzy Set)**.** *An intuitionistic fuzzy set X over a universe of discourse U is given by*

$$X = \left\{ \left(x, \langle \mu_X(x), \nu_X(x) \rangle \right) | x \in U \right\}$$

where $\mu_X : U \to [0,1]$, and $\nu_X : U \to [0,1]$ verify

$$0 \leq \mu_X(x) + \nu_X(x) \leq 1 \quad \forall x \in U.$$

$\mu_X(x)$ and $\nu_X(x)$ represent the degree of membership and degree of non-membership of x in X, respectively.

An intuitionistic fuzzy set becomes a fuzzy set when $\mu_X(x) = 1 - \nu_X(x)\ \forall x \in U$. However, when there exists at least one value $x \in U$ such that $\mu_X(x) < 1 - \nu_X(x)$, an extra parameter has to be taken into account when working with intuitionistic fuzzy sets: the hesitancy degree, $\tau_X(x) = 1 - \mu_X(x) - \nu_X(x)$, that represents the amount of lacking information in determining the membership of x to X. If the hesitation degree is zero, the reciprocal relationship between membership and non-membership makes the latter one unnecessary in the formulation as it can be derived from the former.

Szmidt and Kacprzyk in [26] defined the intuitionistic fuzzy preference relation as a generalisation of the concept of fuzzy preference relation.

Definition 5 (Intuitionistic Fuzzy Preference Relation)**.** *An intuitionistic fuzzy preference relation B on a finite set of alternatives* $X = \{x_1, \ldots, x_n\}$ *is characterised by a membership function* $\mu_B\colon X \times X \to [0,1]$ *and a non-membership function* $\nu_B\colon X \times X \to [0,1]$ *such that*

$$0 \leq \mu_B(x_i, x_j) + \nu_B(x_i, x_j) \leq 1 \;\; \forall (x_i, x_j) \in X \times X.$$

with $\mu_B(x_i, x_j) = \mu_{ij}$ *interpreted as the certainty degree up to which* x_i *is preferred to* x_j*; and* $\nu_B(x_i, x_j) = \nu_{ij}$ *interpreted as the certainty degree up to which* x_i *is non-preferred to* x_j*.*

Notice that in [28] it has been proved that there exists a one-to-one correspondence between the set of reciprocal intuitionistic fuzzy preference relations and the set of asymmetric fuzzy preference relations, and so the Consistency measures above can be directly applied to the case of IRFPRs.

2.3. *Reciprocal Intuitionistic Fuzzy Preference Relations and Asymmetric Fuzzy Preference Relations*

Let us denote with $\mathscr{B}$ the set of reciprocal intuitionistic fuzzy preference relations:

$$\mathscr{B} = \Big\{ B = (b_{ij}) | \forall ij:\; b_{ij} =< \mu_{ij}, \nu_{ij} >,\; \mu_{ij}, \nu_{ij} \in [0,1], \\ \mu_{ii} = \nu_{ii} = 0.5\, \mu_{ij} = \nu_{ji},\; 0 \leq \mu_{ij} + \nu_{ij} \leq 1 \Big\}$$

and with $\mathscr{R}$ the set of fuzzy preference relations

$$\mathscr{R} = \Big\{ R = (r_{ij}) | \forall ij:\; r_{ij} \in [0,1] \Big\}$$

Let us define the following mapping $f\colon \mathscr{B} \longrightarrow \mathscr{R}$

$$f(B) = (f(b_{ij})) = (\mu_{ij}) = (r_{ij}) = R.$$

We have:

- Function f is well defined, i.e. given $B \in \mathscr{B}$ it is true that $f(B) \in \mathscr{R}$.
- Function f is an injection. Indeed, let B_1 and B_2 two reciprocal intuitionistic fuzzy preference relations such that $f(B_1) = f(B_2)$. Then we have that

  $$f(r^1_{ij}) = f(r^2_{ij}) \Leftrightarrow \mu^1_{ij} = \mu^2_{ij} \;\forall i, j.$$

  Because $\mu^1_{ij} = \nu^1_{ji}$ and $\mu^2_{ij} = \nu^2_{ji}$ then it is obvious that

  $$\nu^1_{ij} = \nu^2_{ij} \;\forall i, j.$$

  Therefore we have that

$$b_{ij}^1 =< \mu_{ij}^1, \nu_{ij}^1 >=< \mu_{ij}^2, \nu_{ij}^2 >= b_{ij}^2 \ \forall i, j.$$

Consequently, it is concluded that

$$B_1 = B_2.$$

- Function f is not a surjection as not all fuzzy preference relations $R \in \mathscr{R}$ verify $0 \leq r_{ij} + r_{ji} \leq 1$. Thus the range of function function f is the set of asymmetric fuzzy preference relations.

Summarising:
There exists a one-to-one correspondence between the set of reciprocal intuitionistic fuzzy preference relations and the set of asymmetric fuzzy preference relations.

This result can be exploited to define concepts for an intuitionistic fuzzy preference relation via the equivalent known ones of the associated asymmetric fuzzy preference relation. In particular, a methodology to derive a priority vector for an intuitionistic fuzzy preference relation via its corresponding fuzzy preference relation based on the concept of non-dominance degree and to tackle the issue of incomplete information in intuitionistic fuzzy preference relations in the framework of group decision making was presented in [28].

2.4. Expert's degree of confidence

Given a reciprocal intuitionistic fuzzy preference relation, the hesitancy degrees used to define confidence measures at its three different levels: pair of alternatives, alternatives and relation levels, can be defined as follows:

Definition 6. *Given a reciprocal intuitionistic fuzzy preference relation $B = (b_{ij}) = (\langle \mu_{ij}, \nu_{ij} \rangle)$, the* confidence level associated to the intuitionistic preference value b_{ij} *is measured as*

$$CFL_{ij} = 1 - \tau_{ij},$$

with τ_{ij} being the hesitancy degree associated to b_{ij}.

As noted before in Section 2.2, $\tau_{ij} = 1 - \mu_{ij} - \nu_{ij}$ and therefore we have that $CFL_{ij} = \mu_{ij} + \nu_{ij}$. In other words, when $CFL_{ij} = 1$ ($\mu_{ij} + \nu_{ij} = 1$) then $\tau_{ij} = 0$ and there is no hesitation at all. The lower the value of CFL_{ij}, the higher the value of τ_{ij} and the more hesitation is present in the intuitionistic value b_{ij}.

Definition 7. *Given a reciprocal intuitionistic fuzzy preference relation $B = (b_{ij}) = (\langle \mu_{ij}, \nu_{ij} \rangle)$, the* confidence level associated to the alternative x_i *is defined as*

$$CFL_i = \frac{\sum\limits_{\substack{j=1 \\ i \neq j}}^{n} \left(CFL_{ij} + CFL_{ji} \right)}{2(n-1)}.$$

Because B is reciprocal, we have that $CFL_{ij} = CFL_{ji}$ $(\forall i, j)$ and therefore it is

$$CFL_i = \frac{\sum\limits_{\substack{j=1 \\ i \neq j}}^{n} CFL_{ij}}{n-1}.$$

A similar interpretation of CFL_i with respect to the confidence on the preference values on the alternative x_i can be done as it was done above with CFL_{ij}.

Definition 8. *The* confidence level associated to a reciprocal intuitionistic fuzzy preference relation $B = (b_{ij}) = (\langle \mu_{ij}, \nu_{ij} \rangle)$ *is measured as*

$$CFL_B = \frac{\sum\limits_{i=1}^{n} CFL_i}{n}.$$

Notice that when $CFL_B = 1$, then the reciprocal intuitionistic fuzzy preference relation B is a reciprocal fuzzy preference relation.

3. The proposed approach

In this section we are going to present the confidence based model for intuitionistic fuzzy preference relation. The proposed approach has two main stages the completion stage, and the consensus stage to bring the experts opinion closer. A general overview of the proposed approach is depicted in Fig. 1.

3.1. Completion model

The existing equivalence between the set of asymmetric fuzzy preference relation and the set of intuitionistic reciprocal fuzzy preference relations pointed out in [27] allows to solve problems associated to reciprocal intuitionistic fuzzy preference relations by solving the corresponding problem to their equivalent asymmetric fuzzy preference relations. Before doing this, we present the formal definition of the concept of incomplete preference relation [17]:

Definition 9. *A function $g : X \longrightarrow Y$ is partial when not every element in the set X necessarily maps to an element in the set Y. When every element from the set X maps to one element of the set Y then we have a total function.*

Definition 10. *A preference relation on a set of alternatives with a* partial *membership function is an* incomplete preference relation.

It is assumed that for incomplete reciprocal intuitionistic fuzzy preference relations, given a pair of alternatives (x_i, x_j) for which b_{ij} is not known, both membership and non-memberships will be unknown. Due to reciprocity, we have that if b_{ij} is not known

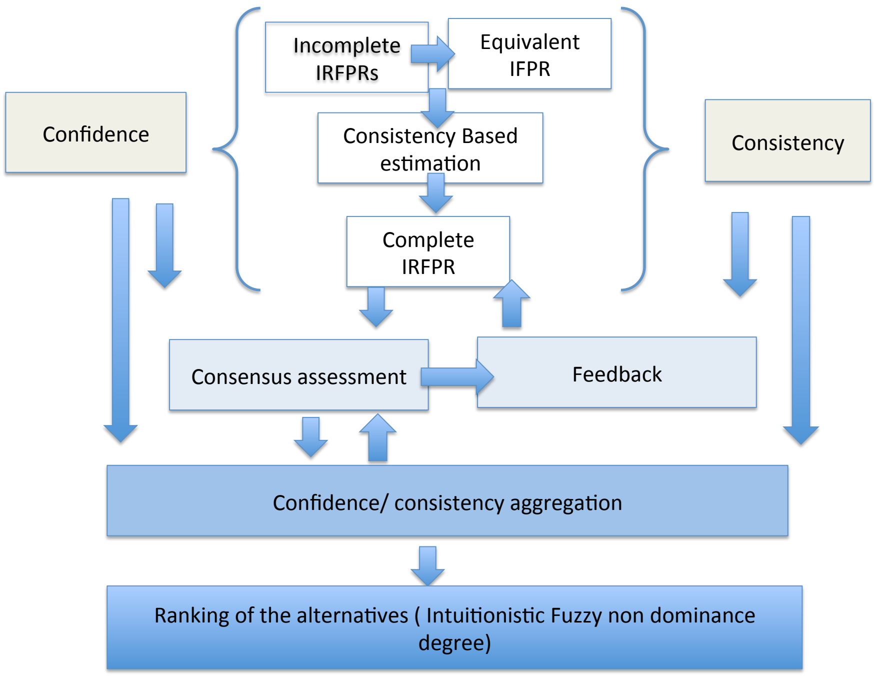

Figure 1. GDM problem resolution steps.

then b_{ji} is also not known. In general the letter x will be used when a particular entry of an incomplete reciprocal intuitionistic fuzzy preference relation is unknown/missing. Thus, if B is an incomplete reciprocal intuitionistic fuzzy preference relation, then $R = F(B)$ will be an incomplete asymmetric fuzzy preference relation and so the missing preference value $r_{ij}(i \neq j)$ can be partially estimated, using an intermediate alternative x_k, with the value:

$$cr_{ij}^{k} = \begin{cases} 0, & (r_{ik}, r_{kj}) \in \{(0,1),(1,0)\} \\ \dfrac{r_{ik} \cdot r_{kj}}{r_{ik} \cdot r_{kj} + (1 - r_{ik}) \cdot (1 - r_{kj})}, & \text{Otherwise.} \end{cases}$$

The following notation is introduced:

$$\begin{aligned} A &= \{(i,j) \mid i,j \in \{1,\ldots,n\} \wedge i \neq j\}; \\ MV &= \{(i,j) \mid r_{ij} \text{ is unknown}, \ (i,j) \in A\}; \\ EV &= A \setminus MV. \end{aligned}$$

MV is the set of pairs of different alternatives for which the fuzzy preference degree is unknown or missing; EV is the set of pairs of different alternatives with known fuzzy preference values. The global multiplicative transitivity based estimated value, cr_{ij}, is defined as follows:

$$cr_{ij} = \frac{\sum_{k \in R_{ij}^{01}} cr_{ij}^{k}}{\# R_{ij}^{01}}$$

where $H_{ij}^{01} = \{k \in R_{ij}^{01} | (i,j) \in MV \wedge (i,k),(k,j) \in EV\}$.

The iterative procedure to complete reciprocal fuzzy preference relations developed in [17] can be applied to complete R and, consequently, to complete the incomplete reciprocal intuitionistic fuzzy preference relation B as the following example illustrates. Notice that the cases when an incomplete fuzzy preference relation cannot be successfully completed are reduced to those when no preference values involving a particular alternative are known. This is the case when a whole row or column is completely missing. Therefore the general sufficient condition for an incomplete fuzzy preference relation to be completed is that a set of $n-1$ non-leading diagonal preference values with each one of the alternatives compared at least once is known.

3.2. Confidence-Consistency based Consensus Model IFPRs

In many decision making processes it could be expected to associate a higher importance degree to the experts that provides both the more coherent or consistent answer and also the ones that present the highest degree of confidence with the provided solutions. In this section we present a new consensus approach that takes both experts degree of confidence and consistency to aggregate the experts opinion. To do so the CC-IOWA operator proposed in [28] to fuse the experts opinions is used. This operator trades off consistency and confidence criteria in both re-ordering the preferences to aggregate and deriving the aggregation weights to use in their fusing to derive the collective preference. Once the collective IFPR is obtained, a proximity index (PI) measuring the level of agreement between the individual and collective preferences is computed. The consensus degree is defined taking into account both the confidence and consistency levels and PI. When the consensus level reaches a threshold value, agreed by the group of experts, the resolution process of the GDM is carried out; otherwise a feedback mechanism is activated, and some personalised recommendations are generated to support the individual experts, until the threshold level of consensus is achieved. The feedback recommendations will help the experts to identify the preference values that should be considered for changing. The recommendations will also include the values the experts should use to increase the level of agreement in a consistent way.

3.2.1. Computing Proximity Indexes

The proximity degrees will measure how close the individual preferences are from the group or collective preferences. The collective preferences are obtained by fusing all the individuals' preferences using the confidence-consistency induced ordered weighted averaging (CC-IOWA) operator:

Definition 11 (CC-IOWA operator). *Let a set of experts, $E = \{e_1, \ldots, e_m\}$, provide preferences about a set of alternatives, $X = \{x_1, \ldots, x_n\}$, using the reciprocal intuitionistic fuzzy preference relations, $\{B^1, \ldots, B^m\}$. A consistency and confidence IOWA (CC-*

IOWA) operator of dimension m, Φ_W^{CC}, is an IOWA operator whose set of order inducing values is the set of consistency/confidence index values, $\{CCI^1, \ldots, CCI^m\}$, associated with the set of experts.

Therefore, the collective reciprocal intuitionistic fuzzy preference relation $B^{cc} = (b_{ij}^{cc}) = (\langle \mu_{ij}^{cc}, \nu_{ij}^{cc} \rangle)$ is computed as follows:

$$\mu_{ij}^{cc} = \Phi_W^{CC}\left(\langle CCI^1, \mu_{ij}^1 \rangle, \cdots, \langle CCI^m, \mu_{ij}^m \rangle\right) = \sum_{h=1}^{m} w_h \cdot \mu_{ij}^{\sigma(h)} \quad (1)$$

$$\nu_{ij}^{cc} = \Phi_W^{CC}\left(\langle CCI^1, \nu_{ij}^1 \rangle, \cdots, \langle CCI^m, \nu_{ij}^m \rangle\right) = \sum_{h=1}^{m} w_h \cdot \nu_{ij}^{\sigma(h)} \quad (2)$$

$$CCI^h = (1-\delta) \cdot CL^h + \delta \cdot CFL^h \quad (3)$$

such that $CCI^{\sigma(h-1)} \geq CCI^{\sigma(h)}$, $w_{\sigma(h-1)} \geq w_{\sigma(h)} \geq 0$ $(\forall h \in \{2, \cdots, m\})$ with $\sum_{h=1}^{m} w_h = 1$, CL_{ij}^h the consistency level associated to $R^h = F(B^h)$, CFL^h the confidence level associated to B^h, and $\delta \in [0,1]$ a parameter to control the weight of both consistency and confidence criteria in the inducing variable.

The general procedure for the inclusion of importance weight values, $\{u_1, \ldots, u_m\}$, in the aggregation process involves the transformation of the values to aggregate under the importance degree to generate a new value and then aggregate these new values using an aggregation operator. In the area of quantifier guided aggregations, Yager provided a procedure to evaluate the overall satisfaction of m important criteria (experts) by an alternative x by computing the weighting vector associated to an OWA operator as follows [35]:

$$w_h = Q\left(\frac{S(h)}{S(m)}\right) - Q\left(\frac{S(h-1)}{S(m)}\right)$$

being Q the membership function of the linguistic quantifier, $S(h) = \sum_{k=1}^{h} u_{\sigma(k)}$, and σ the permutation used to produce the ordering of the values to be aggregated. This approach for the inclusion of importance degrees associates a zero weight to those experts with zero importance degree. The linguistic quantifier is a Basic Unit-interval Monotone (BUM) function $Q : [0,1] \to [0,1]$ such that $Q(0) = 0$, $Q(1) = 1$ and if $x > y$ then $Q(x) \geq Q(y)$.

Yager extended this procedure to the case of IOWA operator. In this case, each component in the aggregation consists of a triple, with first element being the argument value to aggregate, the second element the importance weight value associated to the first element and the third element being the order inducing value [34]. The same expression as above is used with σ being the permutation that order the induce values from largest to lowest. In our case, we propose to use the consistency/confidence values associated with each expert both as an importance weight and as the order inducing values. Thus, the ordering of the preference values is first induced by the ordering of the experts from the most to the least consistent/confident, and the weights of the CC-IOWA operator is obtained as follows:

$$w_h = Q\left(\frac{\sum_{k=1}^{h} CCI^{\sigma(k)}}{T}\right) - Q\left(\frac{\sum_{k=1}^{h-1} CCI^{\sigma(k)}}{T}\right)$$

with $T = \sum_{k=1}^{m} CCI^k$.

The metric used to compute consistency indexes is used here to compute the proximity (similarity) between an individual IFPR, $R^h = (r_{ij}^h)$, and the collective IFPR, $R^c = (r_{ij}^c)$, at the three different levels of the relation:

Level 1. *Proximity index on pairs of alternatives*. The proximity of an expert, e_h, preference value on the pair of alternatives (x_i, x_k) to the group one, denoted PP_{ik}^h, is defined as:

$$PP_{ij}^h = 1 - d(r_{ij}^h, r_{ij}^c)$$

Level 2. *Proximity index on alternatives*. The proximity of an expert, e_h, preferences involving the alternative x_i to the group ones, denoted PA_i^h, is defined as:

$$PA_i^h = \frac{\sum_{j=1; j\neq i}^{n} (PP_{ij}^h + PP_{ji}^h)}{2(n-1)}$$

Level 3. *Proximity index on the relation*. The proximity of an expert, e_h, preference relation to the group one, denoted PI^h, is defined as:

$$PI^h = \frac{\sum_{i=1}^{n} PA_i^h}{n}$$

3.2.2. Computing Consensus Levels

Given an IFPR, *R*, its consensus level (CL) is defined as follows:

$$CL = \delta \cdot CCL + (1 - \delta) \cdot PI \qquad (4)$$

where $\delta \in [0, 1]$ is a parameter to control the weight of both, on the one hand the Consistency-Confidence Criteria and on the other hand the proximity criteria. Similar expressions apply to CL_i and CL_{ij}, respectively. A value of $\delta > 0.5$ is used to provide more importance to the consistency-confidence index in the computation of the consensus degrees. The particular value to use will obviously depend on the group of experts and the importance they would like to allocate to the consistency and the confidence of each expert, but we can assume that the threshold value $\gamma \in [0.5, 1)$.

The consensus levels can be used to decide whether the feedback mechanism should be applied or not to give advice to the experts, or when the consensus reaching process has to come to an end. When CL^h $(h = 1, \ldots, m)$ satisfies a minimum satisfaction threshold value $\gamma \in [0.5, 1)$, then the consensus reaching process ends, and the selection process is applied to achieve the solution of consensus.

3.2.3. Feedback Mechanism

When at least one of the experts' consensus levels is below the fixed threshold value, a feedback mechanism is activated to generate personalized advice to those experts. This activity includes two steps: *Identification of the preference values* that should be changed and *Generation of advice*.

1. **Identification of the Preference Values:**
   The preference values that are contributing less to the consensus are identified. To do that, the following three step identification procedure that uses the proximity and consistency indexes is carried out:

   Step 1. The experts with a consensus level lower than the threshold value γ are identified:

   $$EXPCH = \{h \mid CL^h < \gamma\}.$$

   Step 2. For the identified experts, their alternatives with a consensus level lower than the satisfaction threshold γ are identified:

   $$ALT = \{(h,i) \mid e_h \in EXPCH \ \& \ CL_i^h < \gamma\}.$$

   Step 3. Finally, the preference values to be changed are:

   $$APS = \{(h,i,k) \mid (h,i) \in ALT \ \& \ CL_{ik}^h < \gamma\}.$$

2. **Generation of Advice:**
   The feedback mechanism generates personalized recommendation rules, which will tell the experts the preference values they should change and the new preference values to use in order to increase their consensus level. For all $(h,i,j) \in APS$, the personalised recommendation rules are identified as follow:

   (a) If $(i,j) \in EV^h$ the recommendation generated for expert e_h is: *"You should change your preference value for the pair of alternatives (i,j), $r_{ij}^h = \left\langle \mu_{ij}^h, \nu_{ij}^h \right\rangle$, to a value closer to $rr_{ij}^h = \left\langle r\mu_{ij}^h, r\nu_{ij}^h \right\rangle$."*

   (b) If $(i,j) \in MV^h$ the recommendation generated for expert e_h is: *"Your missing preference value for the pair of alternatives (i,j) should be as close as possible to $rr_{ij}^h = \left\langle r\mu_{ij}^h, r\nu_{ij}^h \right\rangle$."*

$$\left\langle r\mu_{ij}^h, r\nu_{ij}^h \right\rangle = \left\langle \delta \cdot \mu_{ij}^h + (1-\delta) \cdot \mu_{ij}^c, \ \ \delta \cdot \nu_{ij}^h + (1-\delta) \cdot \nu_{ij}^c \right\rangle$$

4. Conclusions

In this contribution we have presented a new consensus driven approach to deal with decision making situations under high level of uncertainty. To do so this model allows the experts to allocate their uncertainty by means of Intuitionistic fuzzy preference relations and is able to estimate the incomplete missing preference relations based on the experts degree of both consistency and confidence. Moreover the proposed approach is able to provide recommendations to the experts to bring their opinions closer taking into consideration the experts degree of confidence with the provided solutions.

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