# Strategic Weight Manipulation in Multiple Attribute Decision Making in an Incomplete Information Context

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***Abstract*—In some real-world multiple attribute decision making (MADM) problems, a decision maker can strategically set attribute weights to obtain her/his desired ranking of alternatives, which is called the strategic weight manipulation of the MADM. Sometimes, the attribute weights are given with imprecise or partial information, which is called incomplete information of attribute weights. In this study, we propose the strategic weight manipulation under incomplete information on attributes weights. Then, a series of mixed 0-1 linear programming models (MLPMs) are proposed to derive a strategic weight vector for a desired ranking of an alternative. Finally, a numerical example is used to demonstrate the validity of our models**

***Keywords—multiple attribute decision making; strategic weight manipulation; ranking; incomplete information***

## I. INTRODUCTION

Multiple attribute decision making (MADM) refers to the problem of ranking alternative based on the evaluation information of alternatives associated with multiple attributes [14, 29]. MADM has been widely used in many fields including engineering, technology, economy, management and military [6, 12, 22, 32].

The attribute weights play an important role in MADM problems. In the existing literature, there are several approaches to obtain the attribute weights from the decision maker's preference information on the set of attributes [1, 7, 15, 18, 25, 27]. However, sometimes, decision makers might find difficult to provide precise attribute weights because of time pressure and limited expertise in regards to the problem domain, which could result in incomplete information of attribute weights.

Decision makers may express their opinions dishonestly to favor their own interest, which is generally known as strategic manipulation or non-cooperative behaviors [11, 13, 23, 26, 30, 31]. Thus, the strategic weight manipulation problem in which a decision maker may strategically set attribute weights in order to obtain her/his desired ranking of the alternatives in the process of setting attribute weights in a MADM problem is worth dealing with.

Although there exists numerous approaches to set attribute weights [1, 7, 15, 18, 25, 27], it is also true that the general theoretical framework that governs the strategic weight manipulation is not always considered. In this study, we define the concept of ranking range of an alternative in MADM under incomplete information of attribute weights. Then, a series of mixed 0-1 linear programming models (MLPMs) are proposed to derive a strategic weight vector by manipulating the decision makers' attribute weights under their given incomplete information.

The rest of this paper is organized as follows. Section 2 provides the basic MADM concepts and the description of the proposed strategic weight manipulation problem under incomplete information of attribute weights. Section 3 proposes mixed 0-1 linear programming models to obtain the ranking range of an alternative and a strategic weight vector to manipulate the ranking of an alternative under incomplete information. Section 4 provides a numerical example that shows the validity of the proposed models. Finally, Section 5 concludes the study.

## II. PRELIMINARIES

This section introduces the basic concepts regarding MADM problem, incomplete information of attribute weights, and the proposed strategic weight manipulation problem as needed for the rest of the paper.

### *A. MADM problem*

Let $X=\{x_1,...,x_n\}$ be the set of alternatives, $A=\{a_1,..., a_m\}$ the set of predefined attributes, and $w=(w_1,w_2,...,w_m)$ the associated weight vector of the attributes, such that $w_j \geq 0$ and $\sum_{j=1}^{m} w_j = 1$.

Let $V=[v_{ij}]_{n\times m}$ be the decision matrix given by the decision maker, where $v_{ij}$ is the preference value for the

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alternative $x_i \in X$ with respect to the attribute $a_j \in A$, representing how well alternative $x_i$ verifies attribute $a_j$. Generally, the resolution process of a MADM problem includes two steps [6, 12, 14, 22, 29, 32]:

(1) Normalization of the decision matrix. The decision matrix $V=[v_{ij}]_{n\times m}$ is transformed into its corresponding standardized decision matrix form $\bar{V}=[\bar{v}_{ij}]_{n\times m}$:

(i) If $a_j \in A$ is a benefit attribute

$$\bar{v}_{ij} = \frac{v_{ij} - \min_i(v_{ij})}{\max_i(v_{ij}) - \min_i(v_{ij})} \quad (1)$$

(ii) If $a_j \in A$ is a cost attribute

$$\bar{v}_{ij} = \frac{\max_i(v_{ij}) - v_{ij}}{\max_i(v_{ij}) - \min_i(v_{ij})} \quad (2)$$

(2) Aggregation of the standardized decision matrix and ranking of alternatives. In this study, we use a weighted average (WA) operator as the decision evaluation function, $D(x_i)$:

$$D(x_i) = WA_w(\bar{v}_{i1}, \bar{v}_{i2}, \ldots, \bar{v}_{im}) = \sum_{j=1}^{m} w_j \bar{v}_{ij} \quad (3)$$

Alternatives are ranked according to their evaluation value. Let $Q_k = \left\{x_i \middle| D(x_i) > D(x_k), i=1,2,\ldots,n\right\}$ be the set alternatives whose decision evaluation value is greater than that of alternative $x_k$, and let $|Q_k|$ be its cardinality. Because $x_k \notin Q_k$, then the following definition of the ranking position of an alternative is justified:

$$r(x_k) = \left|\left\{x_i \middle| D(x_i) > D(x_k), \quad i=1,2,\ldots,n\right\}\right| + 1.$$

Let $r_w(x_k)$ be the ranking of alternative $x_k$ obtained when the weight vector of the attributes $w=(w_1, w_2, \ldots, w_m)$ is used in Eq. (3). Notice that $r_w(x_k)$ can change when the weight vector $w=(w_1, w_2, \ldots, w_m)$ changes, and consequently a manipulation of the weight vector could lead to a change in the ranking order of the alternatives.

*B. Incomplete information of attribute weights*

As mentioned before, in MADM problems, decision makers could provides incomplete attribute weights information, which can be modeled using one of the following representation formats [19, 24]:

- Weak ranking:

$$WR = \{w_i \geq w_j \geq 0; i \neq j\} \quad (4)$$

- Strict ranking:

$$SR = \{w_i - w_j \geq \alpha_i; i \neq j; w_i, w_j \geq 0\} \quad (5)$$

- Ranking multiples:

$$RM = \{w_i \geq \beta_i w_j, i \neq j, w_i \geq 0, w_j \geq 0\} \quad (6)$$

- Interval form:

$$I = \{\gamma_i \leq w_i \leq \gamma_i + \varepsilon_i\} \quad (7)$$

- Ranking differences:

$$RD = \{w_i - w_j \geq w_k - w_i; i \neq j \neq k; w_i, w_j, w_k \geq 0\} \quad (8)$$

- Bounded:

$$B = \{\theta_i \leq w_i / w_j \leq \theta_i + \varepsilon_i : w_i, w_j \geq 0\} \quad (9)$$

In Eqs. (4)-(9), $\{\alpha_i\}$, $\{\beta_i\}$, $\{\gamma_i\}$, $\{\varepsilon_i\}$ and $\{\theta_i\}$ are all assumed to be non-negative.

Let $S$ be the set of attribute weights subject to one or more of the Eqs. (4)-(9). Such set $S$ is referred to as a set of incomplete attribute weights. The following example illustrates three different sets $S$ of incomplete attribute weights.

***Example 1:*** Assume three attributes $\{a_1, a_2, a_3\}$. The following are three possible sets of incomplete attribute weights information:

(1) $S_1 = \{(w_1, w_2, w_3)^T \mid w_1 \geq w_2 \geq w_3\}$;

(2) $S_2 = \{(w_1, w_2, w_3)^T \mid w_1 - w_2 \geq w_3 - w_2\}$;

(3) $S_3 = \{(w_1, w_2, w_3)^T \mid w_1 / w_2 \geq 2, \quad w_1 \geq w_3, \quad w_2 \geq w_3\}$.

*C. The proposed strategic weight manipulation problem*

Because different attribute weights may yield different ranking of alternatives, in the following the ranking range of alternatives for a set of incomplete attribute weights is proposed:

***Definition 1:*** Let $S$ be a set of incomplete attribute weights for a given MADM problem, $\underline{r}(x_k) = \min_{w\in S} r_w(x_k)$ and $\bar{r}(x_k) = \max_{w\in S} r_w(x_k)$ be the best and worst ranking of alternative $x_k$ for all possible incomplete attribute weights in $S$, respectively. Then $R(x_k) = [\underline{r}(x_k), \bar{r}(x_k)]$ is called the ranking range of alternative $x_k$ for attribute weights set $S$.

In a MADM problem, a decision maker may be interested in finding out, and subsequently proposing, the attribute weight vector leading to her/his desired ranking of alternatives. This is referred to as the strategic weight manipulation in MADM under incomplete information of attribute weights set.

## III. STRATEGIC WEIGHT MANIPULATION

In this section, we present mixed 0-1 linear programming models to obtain (A) the ranking range and (B) the strategic attribute weight vector to manipulate the ranking of an alternative.

*A. Ranking range*

Let $y_i \in \{0,1\}$, $M$ be a large enough number, and $D(x_i)$ as defined in Eq. (3). Then, we have:

***Lemma 1:*** (i) $x_i$ $x_k$ if and only if $y_i = 1$ under the conditions $D(x_i) > D(x_k) - (1 - y_i)M$ and $D(x_i) \leq D(x_k) + y_i M$.

(ii) $x_i \precsim x_k$ if and only if $y_i = 0$ under the conditions $D(x_i) \le D(x_k) + y_i M$ and $D(x_i) > D(x_k) - (1-y_i)M$.

The proof of Lemma 1 is provided in Appendix A.

Based on Lemma 1, the following mixed 0-1 linear programming models (10-11) are proposed to obtain the ranking range $R=[\underline{r}(x_k),\overline{r}(x_k)]$ of the alternative $x_k$ a set of incomplete attribute weights $S$:

$$\underline{r}(x_k) = \min \sum_{i=1}^{n} y_i + 1$$
$$s.t. \begin{cases} \sum_{j=1}^{m} w_j \overline{v}_{ij} > \sum_{j=1}^{m} w_j \overline{v}_{kj} - (1-y_i)M, & (i=1,2,...,n) \\ \sum_{j=1}^{m} w_j \overline{v}_{ij} \le \sum_{j=1}^{m} w_j \overline{v}_{kj} + y_i M, & (i=1,2,...,n) \\ y_i = 1 \quad or \quad 0, & (i=1,2,...,n) \\ (w_1, w_2, ..., w_m) \in S \end{cases} \tag{10}$$

and

$$\overline{r}(x_k) = \max \sum_{i=1}^{n} y_i + 1$$
$$s.t. \begin{cases} \sum_{j=1}^{m} w_j \overline{v}_{ij} > \sum_{j=1}^{m} w_j \overline{v}_{kj} - (1-y_i)M, & (i=1,2,...,n) \\ \sum_{j=1}^{m} w_j \overline{v}_{ij} \le \sum_{j=1}^{m} w_j \overline{v}_{kj} + y_i M, & (i=1,2,...,n) \\ y_i = 1 \quad or \quad 0, & (i=1,2,...,n) \\ (w_1, w_2, ..., w_m) \in S \end{cases} \tag{11}$$

*B. Strategic weight manipulation on an alternative*

Let $w_0 = \left(w_1^0, w_2^0, ..., w_m^0\right)$ be the objective weight vector of the attributes in a MADM problem. Let $w=(w_1, w_2, ..., w_m) \in S$ be the decision maker's strategic weight vector to manipulate the ranking of alternative $x_k$.

It is natural that the decision maker wishes to minimize the difference between the objective and strategic weight vectors, i.e.,

$$\min \sum_{j=1}^{m} \left| w_j - w_j^0 \right| \tag{12}$$

Let the decision maker's desired ranking of the alternative $x_k$ be $r^*(x_k)$, then the decision maker would like the following equality to hold

$$r_w(x_k) = r^*(x_k) \tag{13}$$

Based on Eqs (12) and (13), an optimization-based model to find out the decision maker's strategic weight is presented as follows.

$$\begin{cases} \min \sum_{j=1}^{m} \left| w_j - w_j^0 \right| \\ s.t. \quad r_w(x_k) = r^*(x_k) \end{cases} \tag{14}$$

To obtain the optimum solution to model (14), the following notation is introduced: $b_j = w_j - w_j^0$ and $g_j = | w_j - w_j^0 |$.

Model (14) can be transformed in the following equivalent mixed 0-1 linear programming model:

$$\min \sum_{j=1}^{m} g_j$$
$$s.t. \begin{cases} b_j = w_j - w_j^0, & (j=1,2,...,m) \\ b_j \le g_j, & (j=1,2,...,m) \\ -b_j \le g_j, & (j=1,2,...,m) \\ 0 \le g_j \le 1, & (j=1,2,...,m) \\ \sum_{j=1}^{m} w_j \overline{v}_{ij} > \sum_{j=1}^{m} w_j \overline{v}_{kj} - (1-y_i)M, & (i=1,2,...,n) \\ \sum_{j=1}^{m} w_j \overline{v}_{ij} \le \sum_{j=1}^{m} w_j \overline{v}_{kj} + y_i M, & (i=1,2,...,n) \\ \sum_{i=1}^{n} y_i + 1 = r^*(x_k), & (i=1,2,...,n) \\ (w_1, w_2, ..., w_m) \in S \\ \sum_{j=1}^{m} w_j = 1 \\ 0 \le w_j \le 1, & (j=1,2,...,m) \\ y_i = 1 \quad or \quad 0 \end{cases} \tag{15}$$

## IV. NUMERICAL ANALYSIS

Let us consider the following numerical example, with data provided in Appendix B, regarding the Academic Ranking of World Universities (http://www.arwu.org/) to demonstrate the validity of the proposed models. In this example, the set of alternatives consists of 50 universities $\{x_1, x_2, ..., x_{50}\}$, which are ranked using their performance regarding the following set of 6 attributes:

$a_1$ : Quality of Education (Alumni of an institution winning Nobel Prizes and Fields Medals);

$a_2$ : Quality of Faculty 1 (Staff of an institution winning Nobel Prizes and Fields Medals);

$a_3$ : Quality of Faculty 2 (Highly cited researchers in 21 broad subject categories);

$a_4$ : Papers published in Nature and Science;

$a_5$ : Papers indexed in Science Citation Index-expanded and Social Science Citation Index;

$a_6$ : Per capita academic performance of an institution.

First, the standardized decision matrix $\bar{V}=[\bar{v}_{ij}]_{50\times 6}$ is obtained. Let $S=\{(w_1,w_2,...,w_6)^T \mid w_1 \ge w_2 \ ... \ge w_6\}$, then using models (10-11) the ranking range of the alternatives $\{x_1,x_2,...,x_{50}\}$ is obtained, $R$, and given in Table 1.

Table 1: The ranking range $R$ for the 50 universities

| $x_i$ | $R$ | $x_i$ | $R$ | $x_i$ | $R$ |
|---|---|---|---|---|---|
| $x_1$ | [1,1] | $x_2$ | [2,11] | $x_3$ | [3,6] |
| $x_4$ | [2,3] | $x_5$ | [3,5] | $x_6$ | [3,8] |
| $x_7$ | [7,10] | $x_8$ | [7,11] | $x_9$ | [5,10] |
| $x_{10}$ | [5,10] | $x_{11}$ | [10,11] | $x_{12}$ | [12,26] |
| $x_{13}$ | [12,13] | $x_{14}$ | [14,36] | $x_{15}$ | [15,32] |
| $x_{16}$ | [13,16] | $x_{17}$ | [15,28] | $x_{18}$ | [15,22] |
| $x_{19}$ | [15,26] | $x_{20}$ | [14,28] | $x_{21}$ | [24,50[ |
| $x_{22}$ | [22,46] | $x_{23}$ | [16,45] | $x_{24}$ | [14,26] |
| $x_{25}$ | [24,46] | $x_{26}$ | [25,44] | $x_{27}$ | [25,47] |
| $x_{28}$ | [16,25] | $x_{29}$ | [23,32] | $x_{30}$ | [27,44] |
| $x_{31}$ | [19,31] | $x_{32}$ | [16,33] | $x_{33}$ | [24,38] |
| $x_{34}$ | [34,47] | $x_{35}$ | [29,40] | $x_{36}$ | [37,49] |
| $x_{37}$ | [14,42] | $x_{38}$ | [35,47] | $x_{39}$ | [17,40] |
| $x_{40}$ | [39,50] | $x_{41}$ | [33,45] | $x_{42}$ | [29,49] |
| $x_{43}$ | [29,47] | $x_{44}$ | [28,47] | $x_{45}$ | [41,50] |
| $x_{46}$ | [13,44] | $x_{47}$ | [34,49[ | $x_{48}$ | [15,44] |
| $x_{49}$ | [48,50] | $x_{50}$ | [31,49] | | |

Let $w_0=(1/6,1/6,1/6,1/6,1/6,1/6)$ be the objective weight vector of attributes. We consider three cases:

Case A. The incomplete attribute weighs set is: $S=\{(w_1,w_2,...,w_6)^T \mid w_1 \ge w_2 ... \ge w_6\}$;

Case B. The incomplete attribute weighs set is: $S=\{(w_1,w_2,...,w_6)^T \mid w_4 - w_5 \ge w_1 - w_2\}$;

Case C. The incomplete attribute weighs set is: $S=\{(w_1,w_2,...,w_6)^T \mid 0.8 \le w_1/w_2 \le 1.2,\ \ 0.5 \le w_3/w_4 \le 2,$ $0.95 \le w_5/w_6 \le 1.05\}$.

Model (15) is used to manipulate the strategic weight vector $w^*$ so that the desired ranking of different manipulated alternatives, $r^*$, is obtained as listed in Table 2.

Table 2: The strategic weight vector, $w^*$, for desired ranking, $r^*$, of different manipulated alternatives, $x_k$.

| $S$ | $x_k$ | $r^*$ | $w^*$ |
|---|---|---|---|
| Case A | $x_3$ | 3 | (0.250, 0.166, 0.166, 0.166, 0.166, 0.086) |
| | $x_{16}$ | 14 | (0.184, 0.167.0.167.0.167.0.165, 0.150) |
| | $x_{30}$ | 44 | (0.281, 0.281, 0.239, 0.154, 0.045, 0) |
| Case B | $x_3$ | 2 | (0.150, 0.167, 0.197, 0.167, 0.167, 0.153) |
| | $x_{16}$ | 9 | (0.167, 0.080, 0.068, 0.386, 0.3, 001) |
| | $x_{30}$ | 11 | (0, 0, 0, 0.164, 0.164, 0.671) |
| Case C | $x_3$ | 3 | (0.167, 0.147, 0.187, 0.167, 0.167, 0.167) |
| | $x_{16}$ | 14 | (0.181, 0.167, 0.153, 0.167, 0.167, 0.167) |
| | $x_{30}$ | 10 | (0, 0, 0.03, 0.06, 0.444, 0.467) |

## V. CONCLUSION

This paper focuses on some issues on the strategic attribute weight to manipulate the ranking of alternatives for incomplete information of attribute weights. First, we define the concept of the ranking range of an alternative in the MADM, and propose MLPMs to obtain the ranking range of alternatives and design a strategic attribute weight vector to obtain a desired ranking under incomplete information of attribute weights. A numerical example is used to demonstrate the validity of our models. In the future, we argue that it would be worth investigating the MADM strategic weight manipulation with incomplete information in a group decision and consensus reaching context [2, 3, 5, 10, 16, 20], as well as the strategic expert-weight manipulation in the group decision making with preference relations [4, 8, 9, 17, 21].

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## APPENDIX A: PROOF

*Proof of Lemma 1:*

**(i): Necessity:** Suppose that $x_i$ $x_j$ and $D(x_i) \le D(x_j)$. On the one hand, this means that for any $k$ such that $D(x_k) \ge D(x_j)$, we would have that $D(x_k) \ge D(x_i)$, i.e., it would be $Q_j \subseteq Q_i$, and therefore it is $r(x_j) \le r(x_i)$, which contradicts that $x_i$ $x_j$.

**Sufficiency:** If $D(x_i) > D(x_j)$, on the one hand, this means that for any $k$ such that $D(x_k) > D(x_i)$, we would have that $D(x_k) > D(x_j)$, i.e., it would be $Q_i \subseteq Q_j$. On the other hand, it is $x_i \in Q_j$. Because $x_i \notin Q_i$, we have $Q_i \subset Q_j$, and therefore it is $r(x_i) < r(x_j)$, i.e., $x_i$ $x_j$.

**(ii):** The proof is similar to proof of (i).

This completes the proof of Lemma 1.

## APPENDIX B: THE DATA FOR 50 UNIVERSITIES

| $x_i$ | $v_{i1}$ | $v_{i2}$ | $v_{i3}$ | $v_{i4}$ | $v_{i5}$ | $v_{i6}$ |
|---|---|---|---|---|---|---|
| 1 | 100 | 100 | 100 | 100 | 100 | 79.2 |
| 2 | 42.9 | 89.6 | 80.1 | 73.6 | 73.1 | 55.8 |
| 3 | 65.1 | 79.4 | 64.9 | 68.7 | 68.4 | 59 |
| 4 | 78.3 | 96.6 | 51.3 | 56.7 | 67.8 | 58.5 |
| 5 | 69.4 | 80.7 | 55.3 | 71.7 | 61.7 | 69.7 |
| 6 | 53.3 | 98 | 51.3 | 47.2 | 42.9 | 74.4 |
| 7 | 49.7 | 54.9 | 56.2 | 55 | 74.5 | 46.1 |
| 8 | 51 | 66.7 | 39.7 | 57.3 | 43.6 | 100 |
| 9 | 63.5 | 65.9 | 41 | 53.3 | 68.9 | 33.3 |
| 10 | 59.8 | 86.3 | 34 | 42.7 | 50.2 | 44.5 |
| 11 | 47.6 | 50.4 | 44.7 | 58.4 | 62.6 | 37.1 |
| 12 | 29.5 | 47.1 | 58 | 44.5 | 71.4 | 33.4 |
| 13 | 42 | 49.8 | 41 | 47 | 60.5 | 40.9 |
| 14 | 19.2 | 35.5 | 49.2 | 57.8 | 63.5 | 37 |
| 15 | 21.2 | 31.6 | 49.2 | 52.1 | 72.6 | 31 |
| 16 | 37.7 | 33.6 | 38.4 | 47 | 71.9 | 31.1 |
| 17 | 28.1 | 36.2 | 41 | 41.6 | 73.9 | 32.4 |

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|---|---|---|---|---|---|---|
| 18 | 31.6 | 33.8 | 42.3 | 39.4 | 67.7 | 37.8 |
| 19 | 29.5 | 35.5 | 35.5 | 50.2 | 55.6 | 46.1 |
| 20 | 36.3 | 25.3 | 30.8 | 47.5 | 70 | 29.7 |
| 21 | 0 | 39.9 | 37 | 52.1 | 59.3 | 33.5 |
| 22 | 14.5 | 35.8 | 43.5 | 32.9 | 64 | 39.9 |
| 23 | 34.4 | 0 | 51.3 | 41.6 | 76.6 | 25.8 |
| 24 | 34.4 | 24.9 | 51.3 | 42 | 51.7 | 37.2 |
| 25 | 15.4 | 19.2 | 57.1 | 38.9 | 62.1 | 25.9 |
| 26 | 15.4 | 22.1 | 54.3 | 35.6 | 59.6 | 32.8 |
| 27 | 19.9 | 17.2 | 32.4 | 38.2 | 80.1 | 30.3 |
| 28 | 32.8 | 34.8 | 30.8 | 35 | 62.7 | 24.3 |
| 29 | 28.1 | 31.9 | 32.4 | 39.5 | 57.3 | 22 |
| 30 | 21.8 | 18.8 | 32.4 | 36.2 | 65.2 | 41.9 |
| 31 | 29.9 | 36.2 | 30.8 | 33.1 | 55.1 | 29.1 |
| 32 | 31.6 | 37.2 | 27.1 | 31.5 | 58.4 | 23.8 |
| 33 | 29.5 | 16.3 | 39.7 | 32.5 | 64.8 | 24.1 |
| 34 | 15.4 | 18.8 | 42.3 | 32.7 | 64.5 | 27.2 |
| 35 | 18.5 | 32.6 | 37 | 26.4 | 58.4 | 29 |
| 36 | 8.9 | 23.7 | 39.7 | 32.6 | 60.8 | 33.8 |
| 37 | 17 | 59.8 | 27.1 | 41.8 | 19.3 | 40 |
| 38 | 12.6 | 34.1 | 30.8 | 36.8 | 46.2 | 35.1 |
| 39 | 33.6 | 27.4 | 20.5 | 29.7 | 61.9 | 25.3 |
| 40 | 17 | 13.3 | 35.5 | 24.8 | 67.9 | 32.2 |
| 41 | 20.5 | 24.9 | 32.4 | 31.3 | 52.1 | 26.8 |
| 42 | 14.5 | 39.1 | 32.4 | 27.3 | 37.7 | 38.2 |
| 43 | 18.5 | 34.5 | 30.8 | 37.6 | 34.9 | 27.7 |
| 44 | 25.6 | 26.6 | 22.9 | 25.1 | 52.6 | 40.2 |
| 45 | 16.2 | 16.3 | 29 | 37 | 56.3 | 26.6 |
| 46 | 30.3 | 54.3 | 10.3 | 17.6 | 47.9 | 27.7 |
| 47 | 19.9 | 25.3 | 22.9 | 30.6 | 51.8 | 34.9 |
| 48 | 34.8 | 21.6 | 29 | 23.3 | 49.7 | 34.6 |
| 49 | 0 | 31.7 | 35.5 | 23.4 | 53.9 | 26.2 |
| 50 | 21.2 | 21 | 34 | 19.6 | 55.3 | 27.9 |